

SOFT COMPUTING FRAMEWORK FOR THE UNCERTAINTY- BASED OPTIMIZATION OF THE LENGTH AND HEIGHT OF OGEE- CRESTED SPILLWAY

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Abstract

Dams are one of the most important and biggest critical infrastructures of any country. The failure of a dam can be a major catastrophic event, causing irreparable environmental, human and financial losses. The low overflow capacity of the spillway is considered a major failure mode for dams. Generally, the design of spillways is carried out based on deterministic approaches. However, there are many uncertainty factors in the design parameters, which have a crucial influence on the spillway performance. In this study, a new framework is presented for the accurate design of the spillways considering the surrounding uncertainty factors of the effective parameters on spillway failure causes. Therefore, the length and height of an ogee-crested spillway is considered as the design variables to be optimized. For this purpose, a meta-heuristic algorithm based on machine learning techniques is used. This latter consists of the grey wolf optimizer (GWO), while the combination of GWO and the Monte Carlo simulation (MCS) with the Kriging meta-model are utilized as a new framework for the optimum design of spillway under uncertainties. The proposed framework is investigated on the spillway redesign of a real case study in Iran.

Keywords: Dam spillway, machine learning, optimum design, Monte Carlo simulation, grey wolf optimizer, Kriging.

1 INTRODUCTION

Ensuring a safe passing of the water in dams from the upstream to the downstream requires an important class of structures called spillways. These important components have several advantages, such as when the water surface arises and they are also used as controlling tools for the flow managements in water transmission channels [1]. To date, several types of spillways have been constructed based on the shape of these structures, including Ogee Spillways, Drop Spillways, Siphon Spillways, Trough/Chute Spillways, Shaft Spillways, Stepped Spillways and Side Channel Spillways. The choice of the spillways type depends on many factors such as the topographic conditions of the construction site to limit the option number of the crest length and to increase the discharge capacity [2]. Therefore, an inadequate design of these structures can lead to the failure of the dam, which catastrophic consequences. Thus, it is of fundamental importance to accurately design an optimal shape of the spillways taking into account the involved uncertainty [1,2]. The optimal design should also take into consideration the implementation costs of the construction project that account for 20%-80% of the total dam construction cost, while the safety levels of the design should meet the integrity criteria [3,4].

Structural optimization can serve as a suitable framework to solve design problems such as the shape design of spillways or other infrastructures [5]. Among the recent used tools for such purposes is the application of meta-heuristic algorithms. These techniques have been proved to be highly efficient compared to classical gradient-based methods for solving complex and highly nonlinear problems, especially in the civil engineering field. Meta-heuristic algorithms have been successfully used to deal with optimization problems, including, hydraulic-, concrete-, building- and energy-related problems and to optimize machine-learning performance [6–9]. More recently, the structural reliability-based design optimization (RBDO) approaches have shown significant improvements as a viable alternative to deterministic optimization (DO) design method to solve complex optimization problems [10–12]. Using RBDO as a framework for optimal design of structures aims to achieve an accurate optimum design with the consideration of the uncertainty related to the design parameters that are both cost-effective and satisfactory to a certain level of safety.

Based on the above arguments, a new framework using the RBDO concept will be developed in this work by using the benefits of the meta-heuristic algorithms and the machine-learning based reliability approaches. To do so, the Grey Wolf Optimizer (GWO) is utilized as the main optimization approach, while the Monte Carlo simulation (MCS) is used to determine the failure probability of the obtained design. Moreover, to ensure accurate computation, the Kriging (KR) is employed to estimate the performance function response. The proposed framework is applied in a real case to determine the length and height of ogee-crested spillway in Iran.

2 PROBLEM FORMULATION

The main purpose of optimizing the length and height of the ogee-crested spillway, is to reduce the cost associated with the spillway construction. Thus, the optimization problem can be formulated mathematically in terms of the objective function and the associated constraints, according to the literature, as follows [13]:

Objective function:

$$\begin{aligned} Cost = CPI \times [304.71 \times \exp\left(0.5 \times \left(\frac{L - 67.4}{40.72}\right) + \left(\frac{T - 11467}{17069}\right)^2\right) \\ + 0.395 \times \exp(0.036 \times P)] \end{aligned} \quad (1)$$

Constraints:

$$\begin{aligned} Q - Q_{design} > 0 \\ 0 \leq C \leq 3.95 \end{aligned} \quad (2)$$

In the above equations, CPI represents the commodity price index, while L and P denote the length and height of the ogee-crested spillway, respectively. These variables represent the decision variables for the ogee-crested spillway optimization problem, which are the main variables that will be optimized in order to minimize the construction cost and maintain the reliability levels of the ogee-crested spillway. Therefore, the objective function aims to find the optimum values for reducing the cost, while the constraints are for maintaining the safety levels. To that, Q is determined as follows [13]:

$$Q = CLH^{\frac{3}{2}} \quad (3)$$

where C and H denote the discharge coefficient and the head, respectively. During the uncertainty-based optimization process of the length and height of the ogee-crested spillway L , P and H are considered as random variables with normal distribution to take into account the uncertainty related to the design. Thus, in the probabilistic analysis, the used performance function to estimate the probability of failure for ogee-crested spillway can be given as [1]:

$$G(X) = Q - Q_{design} \quad (4)$$

In which X denotes the vector of the previously mentioned random variables

3 PROPOSED METHODOLOGY

The proposed methodology in this work consists of two frameworks based on the meta-heuristic Grey Wolf Optimizer (GWO) Algorithm. The first method utilizes the Deterministic Optimization (DO) approach, in which there is no consideration of the uncertainties (L , P and H are taken as deterministic values) and GWO is used to solve the optimization problem (Eqs. (1) and (2)). The second method is by applying the concept of Reliability Based design Optimization (RBDO). In this framework, GWO will be used to find the optimal values of the design variables at the first stage, then these variables will be treated as random variables, while a data-driven approach, called Kriging technique will be used to reproduce the performance function (Eq. (4)) response. Thereafter, the Monte Carlo Simulation (MCS) will be employed to calculate the failure probability of the system. The following subsections detail briefly the GWO, Kriging and MCS approaches.

3.1 Grey wolf optimizer (GWO)

This algorithm has been introduced by Mirjalili et al [14] by imitating the search and hunting process of grey wolves. In this algorithm, the solutions are divided into: the best solutions,

designed by α , the second one by β , the third by δ , while the rest of the solutions are referred to by ω . Thus, the three best solutions, or in other words the best wolves guide the rest ones during the optimization process. During the hunting process, after the prey is found, α , β and δ wolves will lead the rest of the wolves (i.e., solutions) to pursue and encircle the prey. Thus, the hunting process can be described as follows:

$$\vec{D} = |\vec{C} \cdot \vec{X}_p(t) - \vec{X}(t)| \quad (5)$$

$$\vec{X}(t+1) = \vec{X}_p(t) - \vec{A} \cdot \vec{D} \quad (6)$$

In the above equations, \vec{X} represents a circular configuration of the grey wolf position; \vec{X}_p describes the prey location vector; t denotes the current moment; \vec{A} and \vec{D} are two coefficient vectors that can be given as:

$$\vec{A} = 2\vec{a} \cdot \vec{r}_1 - \vec{a} \quad (7)$$

$$\vec{C} = 2 \cdot \vec{r}_2 \quad (8)$$

where \vec{a} is a vector that decreases linearly from 2 to 0, while \vec{r}_1 and \vec{r}_2 are random vectors with uniform distributions (i.e. between 0 and 1).

The main wolves during the optimization process are assumed to have previous knowledge regarding the prey location, where the next step is the exploitation (hunting) that is achieved when \vec{a} decreases to 0, which means that the wolves are approaching the prey (optimum) location. More details regarding the GWO and avoiding minimum locals can be found in [15].

3.2 Kriging (KR)

The Kriging (KR) is a surrogate technique that is used to solve complex problems with high abilities to describe the relationship between the variables and the outcome response. This approach has been widely used for dealing with various complex problems such as optimization [16,17] and reliability analysis [18–20] problems with high efficiency. This predictive model formulates the performance function $G(\mathbf{X})$ response as follows:

$$G(\mathbf{X}) = \mathbf{f}(\mathbf{X})^T \boldsymbol{\beta} + S(\mathbf{X}) \quad (9)$$

$$\mathbf{f}(\mathbf{X})^T \boldsymbol{\beta} = \beta_1 f_1(\mathbf{X}) + \dots + \beta_k f_k(\mathbf{X}) \quad (10)$$

where the term $\mathbf{f}(\mathbf{X})^T \boldsymbol{\beta}$ represents the regression model, while $\mathbf{f}(\mathbf{X})^T$ is the basic trend function vector and $\boldsymbol{\beta}$ is a regression coefficient vector. $S(\mathbf{X})$ refers to the Gaussian process. For n training points with \mathbf{X}_i ($i=1, \dots, n$) used to construct the KR model, where \mathbf{Y} denotes the vector of responses related to the n training points, the regression coefficient vector and its process variance can be determined using the following formulas:

$$\hat{\boldsymbol{\beta}} = (\mathbf{F}^T \mathbf{R}^{-1} \mathbf{F})^{-1} \mathbf{F}^T \mathbf{R}^{-1} \mathbf{Y} \quad (11)$$

$$\hat{\sigma}_S^2 = \frac{1}{n} (\mathbf{Y} - \mathbf{F} \hat{\boldsymbol{\beta}})^T \mathbf{R}^{-1} (\mathbf{Y} - \mathbf{F} \hat{\boldsymbol{\beta}}) \quad (12)$$

where, \mathbf{R} denotes an $n \times n$ matrix. More details regarding the KR-model can be found in [21].

3.3 Monte Carlo Simulation (MCS)

Monte Carlo simulation (MCS) represents the most widely used reliability approach for solving complex problems in several engineering fields [22–24]. The basis of simulation methods is the production of random samples in accordance with the random variable distributions, where the response of the system is determined for each set of random variables generated. In this method, which was proposed by Metropolis and Ulam, all the possible space produced by the samples is covered, where these random samples are generated based on different statistical distribution functions related to LSF random variables. Thereafter, each possibility is assessed based on each set of samples to calculate the associated probability of failure. The overall probability of the system failure is calculated by dividing the number of states $G(\mathbf{X}) \leq 0$ by the total number of sample sets (Eqs (13) and (14)) [25].

$$P_f = P[g(X_1, X_2, \dots, X_n) \leq 0] = \frac{1}{N} \sum_{i=1}^N I(X_1, X_2, \dots, X_n), \quad (13)$$

where, N is the total number of simulations, $I(X_1, X_2, \dots, X_n)$ is a function defined by:

$$I(X_1, X_2, \dots, X_n) = \begin{cases} 1 & \text{if } g(X_1, X_2, \dots, X_n) \leq 0 \\ 0 & \text{if } g(X_1, X_2, \dots, X_n) > 0 \end{cases}, \quad (14)$$

The proposed RBDO framework is described in Figure 1 and referred to hereafter by GWO-KR-MCS.

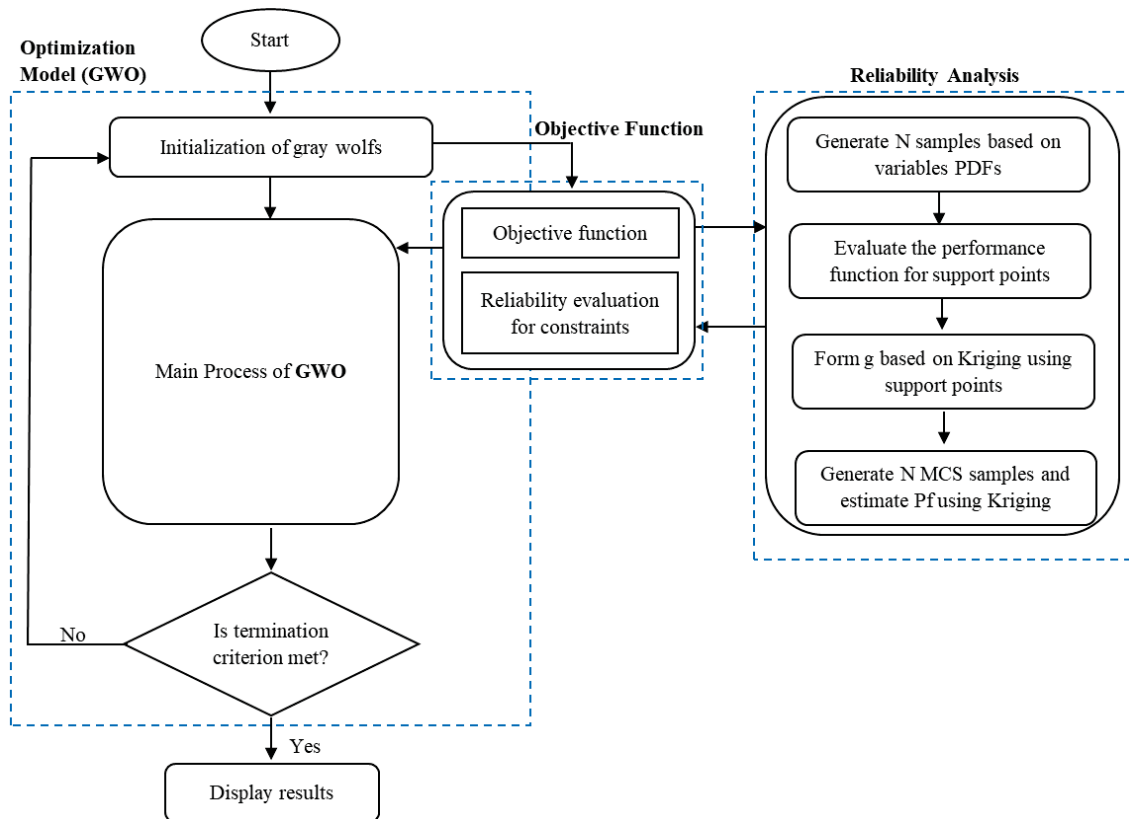


Figure 1: Structure of the proposed RBDO framework.

4 APPLICATION AND RESULTS

4.1 Case study

In order to apply the proposed frameworks in terms of GWO (i.e., DO) and GWO-KR-MCS (i.e., RBDO) a real case study is examined, the spillway of Balarood Dam, located in Iran as illustrated in Figure 2. More information about Balarood Dam is presented in Table 1. This information includes the type, geometries and the volumes of the Dam [13].

Component	Quantity
Dam type	Earth dam with clay core vertical
Dam crest length	1070 m
Dam crest width	10 m
Height from riverbed	75.5 m
Height from foundation	77.5 m
Tank total volume	131 million m ³
50-years-old of sediment	52.39 million m ³

Table 1: Technical Specifications of Ballarood Dam [10].



Figure 2: Geographical location of the Ballarood Dam [26].

4.2 Results and discussion

In this section, the results obtained from the implementation of the proposed frameworks as DO: GWO and RBDO: GWO-KR-MCS are presented. The two proposed frameworks are used to accurately determine the appropriate optimum values of the height and length for the Ogee Crested spillway of Ballarood Dam. Before discussing the results, it is worth mentioning that three statistical indicators are used to estimate the performance of the Kriging-technique for modeling the response of the performance function. These statistical indicators include the mean average percentage error (*MAPE*), standard deviation (*SD*) and coefficient of determination (R^2), which can be expressed as follows [27,28]:

$$MAPE = \frac{100}{n} \times \sum_{i=1}^n \left| \frac{\lambda_{actual}^i - \lambda_{predicted}^i}{\lambda_{actual}^i} \right| \quad (15)$$

$$SD = \sqrt{\frac{1}{n-1} \sum_{i=1}^n \left(\frac{\lambda_{actual}^i - \lambda_{predicted}^i}{\lambda_{actual}^i} \right)^2} \quad (16)$$

$$R^2 = 1 - \frac{\sum_{i=1}^n (\lambda_{actual}^i - \lambda_{predicted}^i)^2}{\sum_{i=1}^n (\lambda_{actual}^i - \lambda_{actual}^{avg})^2} \quad (17)$$

Where λ_{actual}^i and $\lambda_{predicted}^i$ are the i^{th} actual and predicted value of the dam response based on the performance function, respectively. λ_{actual}^{avg} denotes the average of the actual values for the performance function while n is the total number of samples.

Table 2 reports the obtained results that describe the performance of the Kriging model, where low values of MAPE and SD indicate a high performance of the model. The Kriging model yielded a MAPE value of 0.1133 and a SD value of 0.148, which indicate the accuracy of the proposed approach for modeling the response of the performance function. On the other hand, the coefficient of determination R^2 is a powerful indicator for measuring the agreement between the actual and predicted results. Thus, a larger value of R^2 indicates a high performance of the model and that the model is efficient to describe the performance function response. According to Figure 3 and Table 2, the Kriging model manages to give a high value of $R^2=0.985$.

Model	MAPE	SD	R^2
Kriging	0.1133	0.148	0.985

Table 2: Performance metrics of Kriging-model for modeling the performance function response.

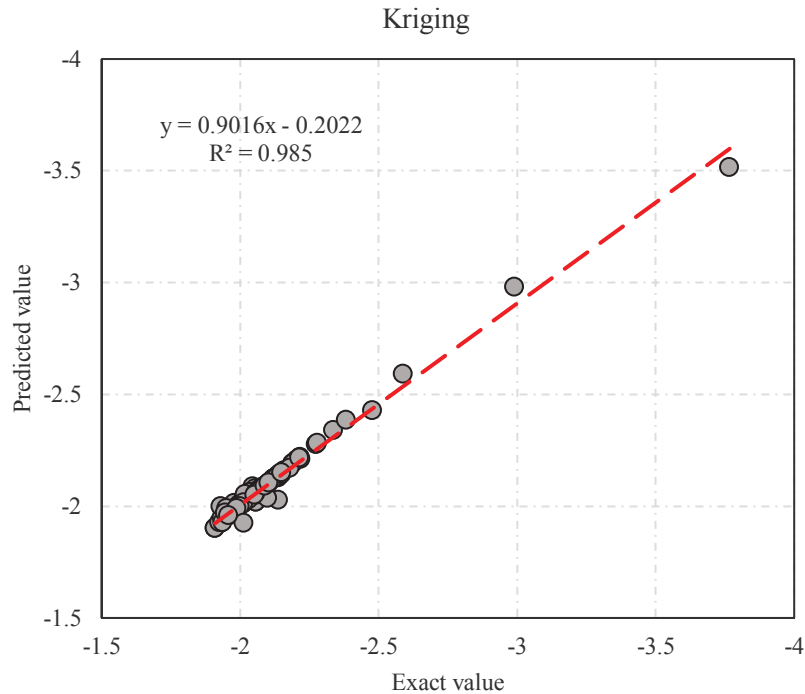


Figure 3: Scatter plots of the performance function predicted by using Kriging model.

The convergence results using GWO-KR-MCS model (i.e., RBDO) and GWO model (i.e., DO) are illustrated in Figure 4. The figure shows that the RBDO approach converges in a slower pace (57 iterations) than the DO approach (8 iterations). These results make sense as the DO approach does not account for any uncertainties in the structure during the computation process unlike the RBDO approach which does. At the same time, these results indicate the robustness of the proposed GWO algorithm as an optimization tool for solving the problem of the Ogee Crested spillway. Table 3 details the optimization results using both proposed frameworks, including the optimum values of the height and the length of the Ogee Crested spillway for Ballarood Dam, the objective function values and the related failure probability estimations. Besides, Table 3 includes the current design information. According to the reported results, the failure probability using the DO-modeling technique is 72%, whereas the current design has a failure probability of 45% given the uncertain parameters and their corresponding distributions. On the other hand, the new RBDO design has the highest design reliability with a failure probability of only 0.01%. Accordingly, the yielded height and length values are 27.8 m and 18.1 m, respectively, with a total cost of $165.1 \times CPI$.

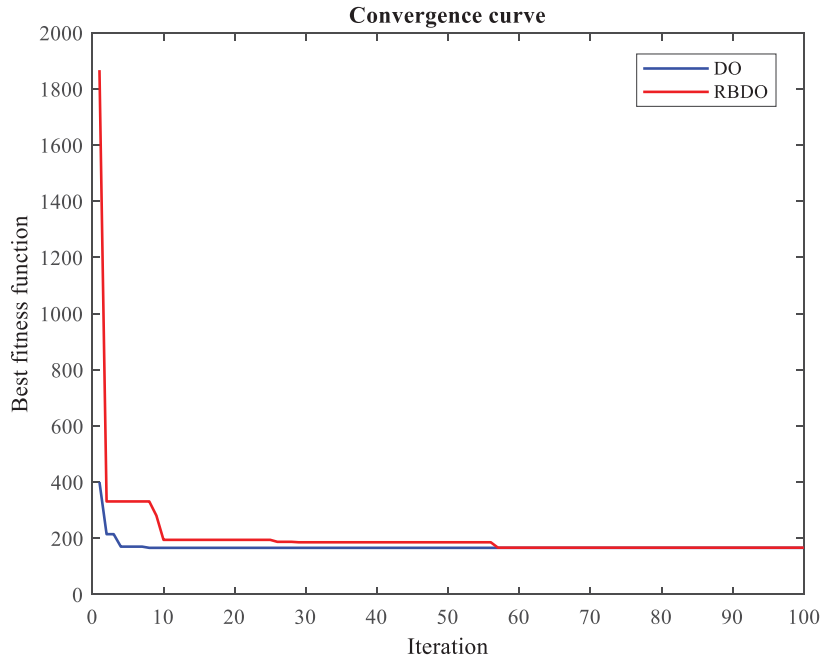


Figure 4: Convergence curves of DO and RBDO approaches.

Design	P (m)	L (m)	Cost	Probability of failure (%)
Current	47.7	20	$201.99 \times CPI$	45
DO	31.47	14.4	$157.75 \times CPI$	72
RBDO	27.8	18.1	$165.1 \times CPI$	0.01

Table 3: Comparison of the proposed approaches and current design.

From the above results, it is clear that the proposed RBDO framework using GWO coupled with KR-MCS method provides the safest design with low failure probability and optimum cost for the Ogee Crested spillway of Ballarood Dam. To that, Figure 5 represents the probabilistic constraint of the discharge under uncertainty. Figure 5 confirms the efficiency of the proposed method, where it is clear that the proposed design using GWO-KR-MCS does not violate the pre-design limit (1950 L/s) indicated by the dashed red line in Figure 5.

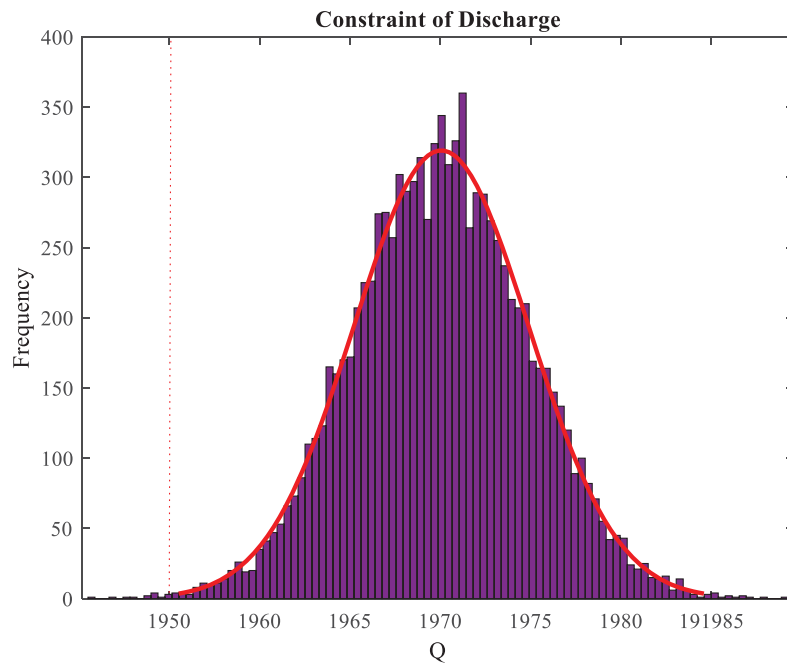


Figure 5: The frequency of discharge under uncertainty of design parameter.

5 CONCLUSIONS

In order to design a safe, reliable and cost-effective Ogee Crested spillway, a new framework using reliability-based design optimization (RBDO) approach was proposed, where the optimal length and height of the ogee-crested spillway for a real case (i.e. Ballarood Dam) is investigated. The proposed method consists of using the Kriging technique as a powerful meta-model to approximate the system response using the performance function. For optimizing the design, the Grey Wolf Optimization (GWO) algorithm is used, while the Monte Carlo Simulation (MCS) is employed to calculate the failure probability of the system under uncertain conditions during the optimization process. The main conclusions that can be drawn based on the results of this paper are the following:

- The proposed Grey Wolf Optimization (GWO) method can be used as an efficient deterministic optimization (DO) approach for optimal shape design of the Ogee Crested spillway.
- The Kriging technique showed an accurate performance in modeling the system response, where the obtained R^2 is 0.985.
- Results indicate that the proposed GWO-KR-MCS framework shows a low failure probability and safe design compared to the DO and current designs.

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