Chapter 7
Numerical Modeling of Historic Masonry Structures

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ABSTRACT

The majority of historical and heritage structures around the world consist of unreinforced masonry walls. A masonry structure is composed of masonry units, such as brick or marble blocks, with or without a joint filling material, such as mortar. A masonry with a joint material is usually made of two different materials (i.e. masonry units and mortar), representing a non-homogeneous and anisotropic structural component. In other words, masonry is a discontinuous structural component whose deformations and failure mechanism are governed by its blocky behavior. Some ancient masonry structures, such as ancient columns and colonnades, are constructed without any form of joint material between the individual blocks. Therefore, the isotropic elastic continuum-based models are not suitable for the simulation of the real nonlinear behavior of masonry walls under applied load.

1. INTRODUCTION

Numerical modeling and analysis of masonry structures is one of the greatest challenges faced by structural engineers. This difficulty is attributed to the presence of joints as the major source of weakness, discontinuity and nonlinearity as well as the existence of uncertainties in the material and geometrical properties. A suitable numerical model must take into account two types of structural behavior: (1)

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behavior of masonry units (i.e. bricks); and (2) behavior of the joint material (i.e. mortar). In the case of ancient masonry structures without any joint material, the numerical model should be able to take into account complicated rocking and sliding of the individual blocks, which may arise during dynamic loadings, such as earthquake excitations.

Numerical modeling strategies of masonry structures are divided into two distinct categories; micro-modeling and macro-modeling. In macro-modeling, a masonry element can be represented using a continuum homogenized model, usually with the finite element method, considering implicitly the effects of mortar joints. In micro-modeling of masonry elements with joint material, the interaction between masonry bricks along the joints is taken into account explicitly using interface elements and utilizing a numerical model such as discontinuous finite element models (D-FEM), discrete/distinct element methods (DEM), discontinuous deformation analysis (DDA), particle flow code (PFC) and finite-discrete element model (FDEM). In the literature, there are extensive researches on the numerical modeling and analysis of masonry structures with their particular advantages and disadvantages.

In this chapter, a thorough overview of the different numerical models proposed for the analysis of masonry structures is presented, while the advantages and disadvantages of each model are pointed out. Furthermore, some comparative studies available in the literature are presented to identify the capabilities and limitations of each computational model introduced throughout this chapter. Finally, some of the general-purpose and specialized finite/discrete element commercial software packages available for numerical modeling and analysis of discontinuous masonry structures are introduced and their advantages and disadvantages are discussed.

Historic structures can be defined as "existing structures with significant cultural value to the society". They can be buildings, towers, bridges, etc. They are mostly made of masonry and timber, sometimes with elements of steel, iron or even concrete (for the more recent constructions). Developed societies ascribe cultural significance to these structures, which are perceived as cultural heritage to the society. Cultural significance means the importance of a site or structure as determined by the aggregate of the values attributed to it. The values considered in this process include those held by experts – the art historians, archaeologists, architects, and others – as well as other values brought forth by new stakeholders or constituents, such as social and/or economic values.

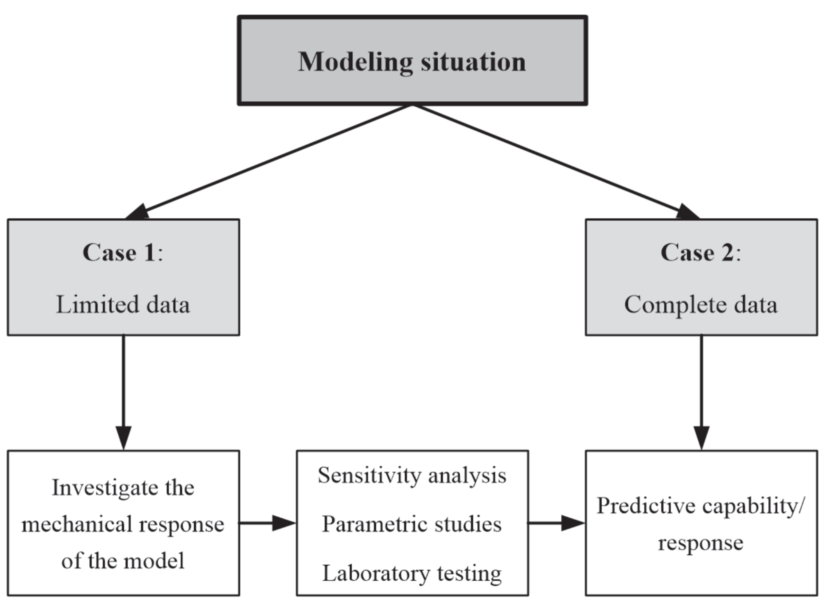
Existing structures and buildings are subjected to the process of degradation with time, which leads to a situation in which they become unable to fulfill the purpose for which they were built. Sometimes, there is also a need to improve the conditions offered by existing buildings or to adapt them to new functions. Furthermore, despite living in an environment of rapid technological evolution, as the society progresses, people realize the necessity and responsibility to maintain the existing architectural heritage and pass it on to future generations.

Degradation of historic/heritage structures made of masonry can occur in different ways. Earthquakes constitute a significant cause of degradation. Any major earthquake can result in losing a number of historic structures. The seismic assessment and rehabilitation of heritage buildings and monuments has become an issue of great importance around the world today. It is the consequenceof the desire to improve existing buildings for adverse conditions, and also of the recognition of the importance of the conservation of our architectural heritage. Rehabilitation of masonry buildings comprises a way of sustainable development and also an act of cultural significance.

Masonry is one of the oldest building materials. It is composed of masonry units (i.e. brick, blocks) with or without mortar. In spite of its simplicity of construction, masonry is a complex material to model. At low levels of stress, masonry behaves as a linear elastic material. Its behavior becomes increasingly non-linear as the load applied on it increases. Modeling old and deteriorated masonry structures involves special considerations and a design philosophy different from that followed for design with fabricated materials such as steel and concrete. Analysis and design of old and deteriorated masonry structures must be achieved with little site-specific data and awareness that deformability and strength properties of both brick and mortar may vary considerably. For example, it is practically impossible to obtain complete data of boundary stresses for each individual brick of an old masonry arch bridge or it is impossible to measure accurately the strength of each mortar joint. Thus, since input data necessary for design predictions of old and deteriorated masonry are limited, it is recommended that a few simple (in geometry) numerical model should be developed primarily by the modeler in order to understand the dominant mechanisms affecting the behavior of the masonry composite system under different material properties and loading configurations. It would be appropriate if these models are validated against controlled laboratory tests or *in situ* tests carried out (such as flat jacks) and static or dynamic monitoring. Once the behavior of the system is understood and the results are validated with that of the laboratory, then it is possible to develop simple idealizations for the predictive capability and response of more complicated structures such as old and deteriorated masonry arch bridges. In addition, the use of sophisticated methods of analysis and computational models may require material parameters that are difficult to be obtained from experimental testing; some of these material parameters may have only a mathematical significance. The engineering judgment and the modeler's experience it is necessary in such cases. The spectrum of modeling situations is presented in Figure 1.

Over the years, extensive research on theoretical methods accompanied by laboratory and field testing has been carried out. In the following section, a review of the current strategies for modeling structural masonry will be given *with an emphasis on those considered to be appropriate for modeling low bond strength* or dry joint masonry.

Figure 1. Spectrum of modeling situations

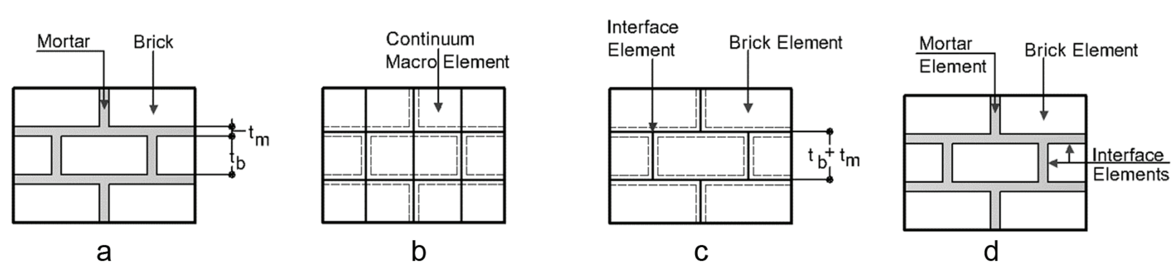


2. STRATEGIES FOR MODELING MASONRY

Numerical modeling of structural masonry is one of the most complicated problems in structural engineering research and practice. This complexity is attributed to the great number of influencing factors such as: dimension and anisotropy of the bricks, joint width and arrangement of bed and head joints, material properties of both brick and mortar, and quality of workmanship. According to Lourenco (2002),Asteris and Tzamtzis (2003), and Asteris et al. (2013 & 2014), the different analytical procedures could be summarized in the following three levels of refinement for masonry models:

- *Modeling masonry as one-phase material (Macro-modeling).* In this strategy, units, mortar and the unit–mortar interface are smeared out in a homogeneous continuum (Figure 2(b)). In other words, no distinction between the individual units and joints is made, and masonry is taken into account as a homogeneous, isotropic or anisotropic continuum medium. While this procedure may be preferred for the analysis of large scale masonry structures, it is not suitable for the detailed stress analysis of a small masonry panel, due to the difficulty of capturing all its expected failure modes. The influence of existing mortar joints as the major source of weakness and nonlinearity cannot be addressed using this strategy.
- *Modeling masonry as two-phase material (Simplified micro-modeling).* In this strategy, the bricks are represented as fictitious expanded bricks by continuum elements with the same size as the original bricks dimensions plus the real joint thickness. The mortar joint is also modeled as an interface with zero thickness as shown in (Figure 2(c)). The interface's stiffness is deduced from the stiffness of the real joints. According to this procedure, the properties of the mortar and the unit/mortar interface are lumped into a common element, while expanded elements are used to represent the brick units. This approach leads to the reduction of the computational effort and yields a model that is applicable to a wider range of structures.
- *Modeling masonry as a three-phase material (Detailed micro-modeling).* In this strategy, units and mortar in the joints are represented by continuum elements whereas the unit–mortar interface is represented by discontinuum elements (Figure 2(d)). While this modeling leads to more accurate results, the level of refinement means that the corresponding analysis is computationally intensive, limiting its application to small scale laboratory specimens and structural details. Sutcliffe et al. (2001) and Asteris and Tzamtzis (2003) have proposed simplified micro-modeling procedures to overcome this problem.

Figure 2. Masonry modeling strategies: (a) Masonry sample; (b) One-phase macro-modeling; (c) Two-phase micro-modeling; (d) Three-phase micro-modeling



3. NUMERICAL MODELING APPROACHES FOR STRUCTURAL MASONRY ANALYSIS: STATE OF THE ART

There are various numerical modeling approaches with different accuracy for the analysis of masonry structures in the literature, as shown in Figure 3. These approaches are discussed in detail in the following subsections.

3.1 Macro-Modeling

3.1.1 Macro-Modeling: Continuum Approaches

As it was pointed out earlier in Sec. 2, in this approach there is no distinction between individual masonry units and mortar in the joints. In other words, a masonry wall is treated as a homogeneous continuum medium. Figure 4 shows results of a non-linear dynamic analysis of a stone masonry building using the macro–modeling approach (Mitsopoulou and Papaloizou 2006). The study evaluates the overall response of an unreinforced stone masonry building under earthquake excitation, as well as the effectiveness

Figure 3. Different numerical modeling approaches for structural masonry analysis

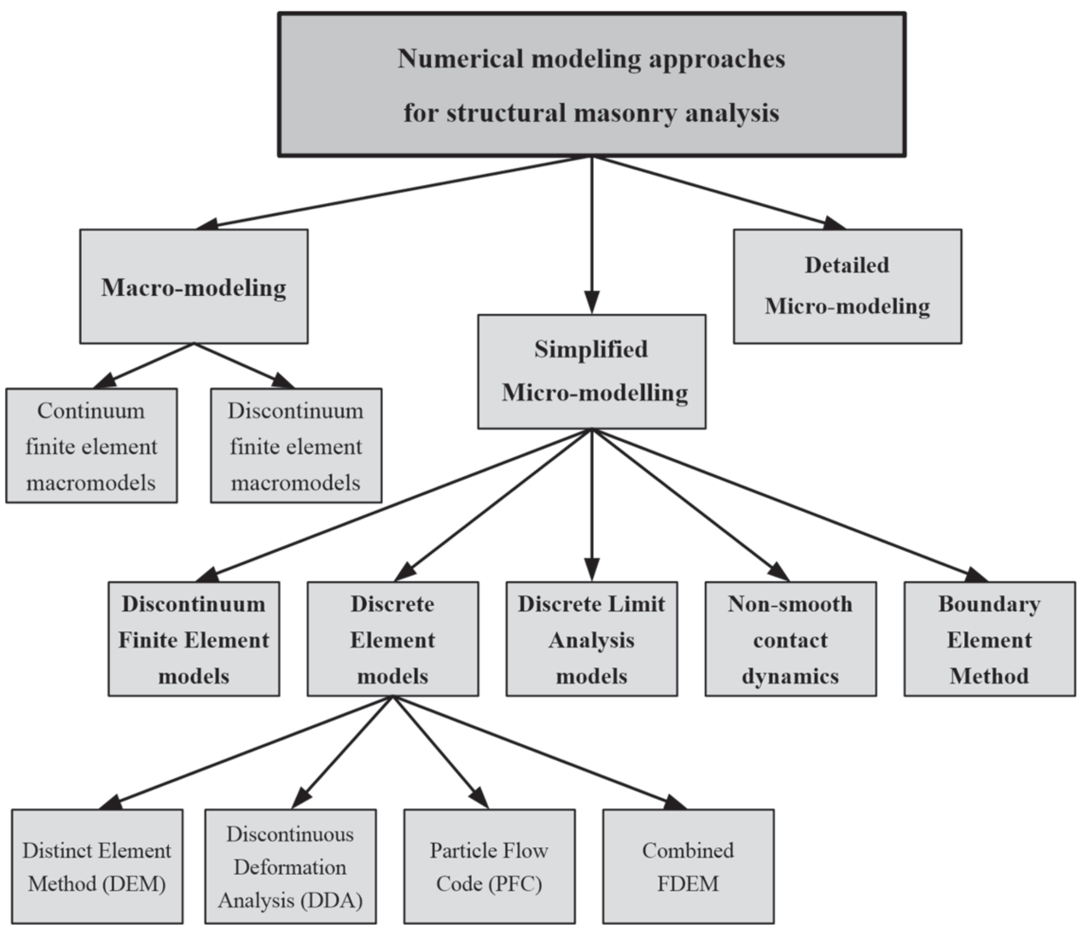
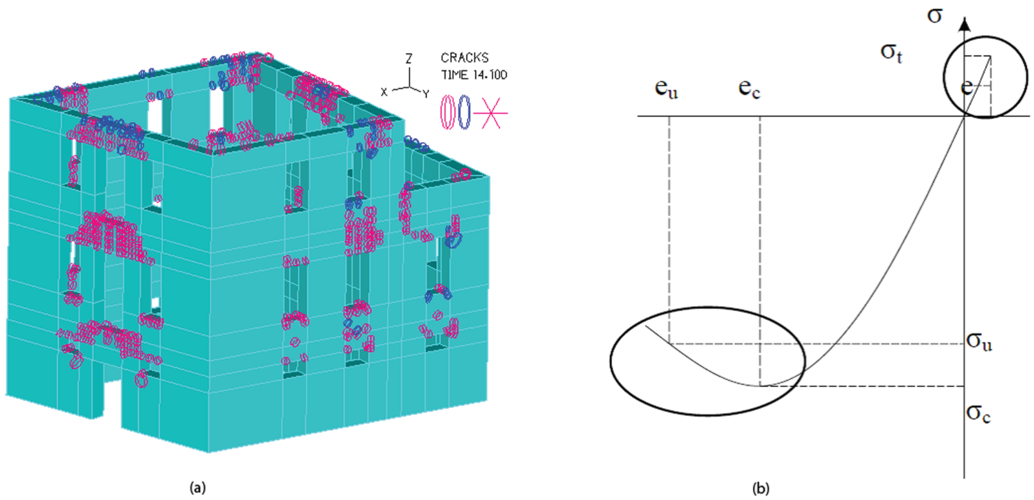


Figure 4. Non-linear dynamic analysis (macro-modeling) of a stone masonry building: (a) Crack pattern; (b) Material model (Mitsopoulou and Papaloizou, 2006)



of various rehabilitation measures, such as diaphragms and bond beams, using the commercial FEM software package ADINA for the analyses. A complex material model is used (Figure 4(b)) so that the overall behavior of the masonry units with high compression strength and the connecting mortar with low tensile strength is sufficiently expressed. The results in Figure 4(a) show cracks that formed during the analysis at various weak areas of the building, such as points below and above wall openings.

Computational models based on continuum plasticity (Lourenço et al. 1998) and continuum damage (Papa et al. 2000) have been developed and implemented successfully for in-plane analysis of masonry shear walls. Lourenço et al. (1998) developed a homogeneous anisotropic continuum model for the analysis of masonry as a composite. In this model, the behavior of the composite media is stated based on average stresses and strains, assuming different elastic and inelastic properties along the material axes. Although this model is suitable for large-scale masonry structures, however, it may be invalid for small-scale masonry walls or masonry structures with large masonry units in which the blocky behavior is the governing behavioral mode.

El-Dakhakhni et al. (2006) developed a macro-model for in-plane analysis of concrete masonry walls with and without reinforcements. In this multi laminate model, a masonry wall is simulated by an equivalent homogeneous media consisting of two sets of planes of weakness along the head and bed joints. To determine the global behavior of the model, the influence of these planes of weakness is smeared. This modeling technique allows prediction of the different possible failure modes, whether the planes of failure follow the mortar joints or not (El-Dakhakhni et al. 2006). The advantage of this model is that, it can predict the initiation and progress of different failure modes (i.e. head joint, bed joint or homogeneous media failure) in a separate or a combined manner.

Asteris et al. (2014) present a methodology for the earthquake resistant design or assessment of masonry structural systems, using case studies from historical masonry structures in the European area. The methodology is based on the damage evaluation of masonry. In this study, damage is estimated using a cubic polynomial function, while the failure criterion is expressed in a non-dimensional form -in order to be able to be applied to other types of masonry materials - as follows (Syrmakezis and Asteris 2001; Asteris and Syrmakezis 2009; Asteris 2010; Asteris 2013):

$$\begin{aligned} f\left(\sigma_x,\sigma_y,\tau\right) = 17.15\left(\frac{\sigma_x}{f_{wc}^{90^\circ}}\right) + 74.57\left(\frac{\sigma_y}{f_{wc}^{90^\circ}}\right) + 32.71\left(\frac{\sigma_x}{f_{wc}^{90^\circ}}\right)^2 + 75.34\left(\frac{\sigma_y}{f_{wc}^{90^\circ}}\right)^2 + 356.74\left(\frac{\tau}{f_{wc}^{90^\circ}}\right)^2 - \\ 17.12\left(\frac{\sigma_x}{f_{wc}^{90^\circ}}\right)\left(\frac{\sigma_y}{f_{wc}^{90^\circ}}\right) + 4.13\left(\frac{\sigma_x}{f_{wc}^{90^\circ}}\right)^2\left(\frac{\sigma_y}{f_{wc}^{90^\circ}}\right) + 1.35\left(\frac{\sigma_x}{f_{wc}^{90^\circ}}\right)\left(\frac{\sigma_y}{f_{wc}^{90^\circ}}\right)^2 + \\ 122.46\left(\frac{\sigma_x}{f_{wc}^{90^\circ}}\right)\left(\frac{\tau}{f_{wc}^{90^\circ}}\right)^2 + 202.20\left(\frac{\sigma_y}{f_{wc}^{90^\circ}}\right)\left(\frac{\tau}{f_{wc}^{90^\circ}}\right)^2 = 1 \end{aligned} \tag{1}$$

where the uniaxial vertical to the bed joints compressive strength is denoted with the symbol $f_{wc}^{90^\circ}$. The applicability of the method is checked via analyses of existing masonry buildings in three countries, namely Greece, Portugal and Cyprus, with different seismicity levels, influencing the risk impacting the masonry structures. Useful conclusions were drawn regarding the effectiveness of the intervention techniques used for the reduction of the vulnerability of the case-study structures, through the comparison of the results obtained. It was shown that the proposed approach offers a ranking method, which helps civil authorities optimize decisions on choosing, among a plethora of structures, which ones present the higher levels of vulnerability and are in need of immediate strengthening, while also helping practicing engineers choose the optimal repairing scenario among a number of competing scenarios.

3.1.2 Macro-Modeling: Discontinuum Approaches

Caliò et al. (2012) proposed an innovative so-called discrete-element model to simulate the nonlinear seismic behavior of masonry buildings. To this end, the in-plane nonlinear response of masonry walls was approximated by an equivalent discrete element using the concept of macro-element discretization. The equivalent macro-element is modeled by the use of an articulated quadrilateral with surrounding rigid edges and to simulate the shear behavior of masonry, two internal diagonal springs are utilized. The flexural and sliding shear behavior is also simulated by discrete distributions of springs in the sides of the quadrilateral that preside over the interaction with the adjacent macro-elements (Caliò et al., 2012). The advantage of the model is that it requires low computational resources for investigating the nonlinear behavior of Unreinforced Masonry (URM) buildings. The accuracy of the proposed model has been investigated by means of analysis of some experimentally tested masonry walls and buildings in the literature. The load-displacement curves and failure patterns obtained from the analyses showed a fairly good agreement with the experimental results in terms of the cyclic response, stiffness, strength and dissipative behavior, as well as damage distribution.

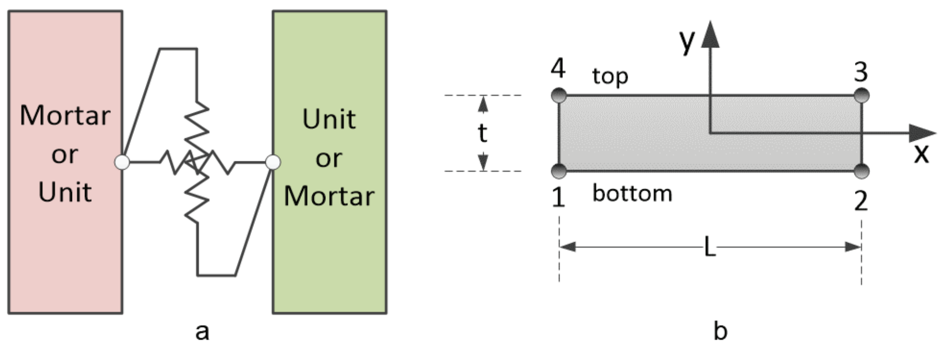
Also, Casolo (2004 & 2009) have also developed a model which considers the heterogeneous solid material as a mechanism consisting of rigid masses connected by elastic-plastic springs (RBSM). The core of this model is a" unit cell "defined by four mass elements connected to each other by two normal springs plus one shear spring at each side. This unit cell is a discrete mechanism that approximates the macroscopic behavior of the heterogeneous masonry material. This model can account for some micro-structure features of the composite material, while it is designed to work at the macro-scale. The discrete element size should be of the same order or larger than the unit cell, i.e. equal or larger than the representative volume element of the masonry. The elastic characteristics of the springs are defined by a specific procedure of identification with the objective to transfer some characteristics of the internal texture to the macro-scale model.

3.2 Simplified Micro-Modeling

3.2.1 Discontinuum Finite Element Models (D-FEMs)

While modeling masonry with a discontinuum finite element model, the mortar joints are represented as discontinuities where a potential crack, slip or crushing failure can occur. The boundary conditions that exist at the unit-mortar interface can be modeled using interface elements that are zero-thickness finite elements (FE) characterized by two surfaces connected to each other that detach in the deformed configuration (Page 1978; Oliveira 2003; Chakrabarty 2006;VoyiadjisandKattan 2005). The function of these elements is to represent the interaction between deformable structures, along surfaces where separation and sliding may arise. The modeling of these boundary conditions is one of the main tasks for all the modeling techniques for masonry structures that are the literal discontinuous structural systems. Despite the large activity during the last five decades in this research field, this remains one of the major issues within the research community dealing with the modeling of masonry structures. The first model, taking into account the behavior of the interfaces, was proposed by Mallick and Severn in 1967 for the modeling of infilled frames' behavior under static loading with special emphasis on the modeling of infill wall-surrounding frame interface. They implemented an interface scheme using a finite element model in which additional contact forces were introduced in those zones where the infill panel-surrounding frame was closed. Several researchers, notably Franklin (1970); King and Pandey (1978); Dawe and Young (1985); Dawe et al. (2001); Mohebkhah et al. (2008); Kouris and Kappos (2011), used spring elements to connect the boundary nodes of the infill panel and the surrounding frame. These elements enable two adjacent nodes to be held together or released according to specified conditions. Each node of an element has two translational degrees of freedom. The element is able to transfer compressive and bond forces, but is incapable of resisting tensile forces. Large values of the normal and tangential stiffnesses are adopted when the link is active. Conversely, the link is released by setting these values to zero. Figure 5(a) illustrates schematically the characteristics of the spring element. More accurate modeling of the interaction between the mortar and units can be achieved by using interface elements, notably Franklin (1970); King & Pandey (1978); Dhanasekar and Page (1986); Liauw and Lo (1988); Schmidt (1989); König (1991); Mosalam et al. (1993);Casolo & Sanjust (2007);Asteris &Tzamtzis (2003); Stavridis and Shing (2010); Acito et al. (2014); Giamundo et al. (2014);Sarhosis &Tsavdaridis (2014);Michel et al. (2014), as shown in Figure 5(b). In software packages, the behavior of interface elements in a model can be governed by a discrete crack model, a Coulomb friction model, or a model based on research by Lourenco (1996) and Van Zijl (2000), which combines Coulomb friction, tension cutoff, and a compression cap. Each of these model components can be specified to undergo hardening and softening. The method proposed by Caliò et al. (2014) has been recently applied to infilled frames as both a macro and micro-modeling approach. A macro-modelling approach for the seismic assessment of Infilled Frame Structures (IFS) is by Calio and Panto (2014). The interaction between frame and infill is

Figure 5. Models for mortar-unit Interface: (a) Spring elements; (b) Interface element



simulated through an original approach in which the frame members are modelled by means of lumped plasticity beam–column elements while the infills are described by plane macro-elements.

Nowadays research is mainly concentrated on the determination of the parameters used to describe the behavior of interface elements. The elastic stiffness parameters that are required to describe the interface elements are often set extremely high to reproduce a rigid behavior. Recently it has been stated by Ciancio et al. (2014), that such an approach leads to false results (Ciancio et al. 2014). In order to solve this problem, the researchers proposed a new procedure, consisting of adding some non-dimensional coefficients (purely geometry-dependent) to the values of the interface stiffness parameters. Thus, accurate tractions at the interface nodes can be recovered.

Lourenço (1996) developed a composite constitutive model for the static analysis of interface elements based on the theory of plasticity. The composite interface model includes a tension cut-off for tensile failure (mode I), a Coulomb friction envelope for shear failure (mode II) and a cap mode for compressive failure.

The structural element used by Lourenço in his study was a shear wall with an opening; see Figure 6(a) and Figure 6(b). The numerical model was checked both qualitatively and quantitatively against experimental data and a high degree of correlation was found as shown in Figure 7. A complete discussion of the numerical results has been reported elsewhere (Lourenço et al. 1997; Lourenço 1998a).

Figure 6. Result of the deformed mesh analysis at a displacement of: (a) 1mm; (b) 25mm(Lourenço, 1996)

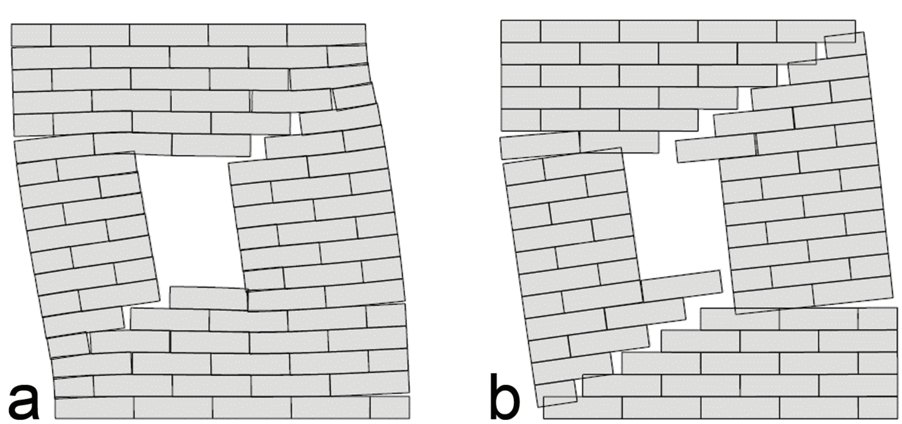
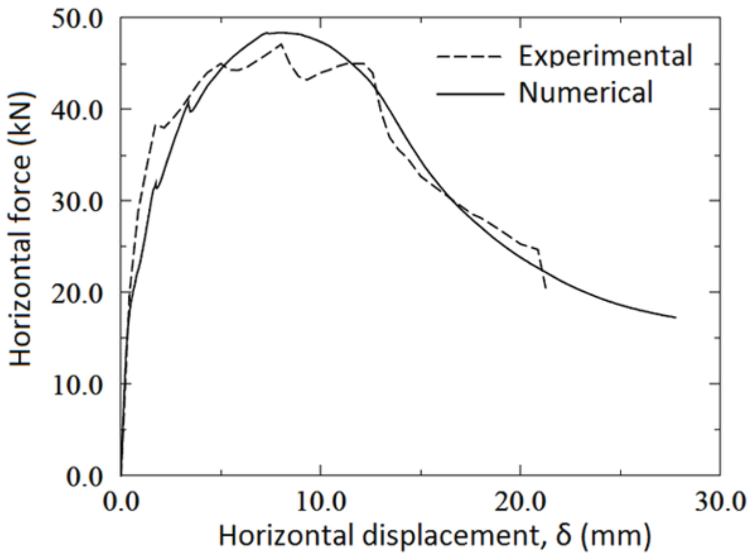


Figure 7. Load – displacement diagram (Lourenço, 1996)



In contrast, Gambarotta and Lagomarsino (1997a) developed a constitutive model for mortar joints based on the principles of damage mechanics. Their model considers both mortar damage and de-bonding of the brick- mortar interface which takes place when opening and frictional sliding are activated. The inelastic strain components (i.e. sliding and opening of the mortar joints) are assumed to be linearly dependent on the mean stress and a damage variable related to the damage mechanics approach. Sliding of the units is limited by the presence of friction at the brick-mortar interface. This model has been applied to the analysis of brick masonry walls under constant vertical loads and horizontal in-plane cyclic forces. Although this approach has been found to simulate the inelastic behavior of masonry (i.e. opening and sliding of joints), it was too computationally expensive to be used for the analysis of full scale masonry wall panels.

Pegon et al. (2001) carried out the analysis of a historic masonry structure. They used interface elements ruled by an improved elasto-plastic Coulomb friction law with material softening and an allowance for dilatancy. The analysis was performed both in 2D and 3D using the computer code CASTEM 2000 (Millard 2003). The 2D model was found to predict deformation and damage patterns quite accurately and, as a result, it was considered to be useful to assess the stability and damage of other structures under combined vertical and lateral loads. The 3D analysis provided an improved description of the behavior of the column of the structure and was used to calibrate the material parameters for the joints found from the 2D analysis. In spite of the apparent success, Pegon et al. (2001) considered that the 3D modeling proved to be too time consuming especially when it was used to carry out a series of parametric studies. Difficulties were also experienced when attempting to mesh the stone blocks and joints.

Van Zijl (2004) developed a computational discrete crack model to simulate masonry shearing dilatancy behavior appropriately. The dilatancy formulation, which was implemented in the Coulomb-friction part of an interface constitutive law, accounts for dilatancy dependence on the confining pressure, as well as shear-slipping along masonry joints. This model was used successfully for the analysis of some vertically confined masonry shear walls under horizontal in-plane forces. It was also found that employing a constant dilatancy coefficient may lead to the danger of overestimating the masonry shear wall resistance.

Lourenço et al. (2005) used the multi-surface interface model proposed by Lourenço and Rots (1997) to simulate nonlinear behavior of dry joint stone masonry walls under in-plane combined loading. In that study, 8-node plane stress elements with Gauss integration were used to model the stone units, while six-node zero-thickness line interface elements with Lobatto integration were utilized to model the joints. Results from the numerical simulation and experimental tests on dry joint masonry walls were compared, indicating a good correlation between them.

Chaimoon and Attard (2007) proposed a simplified micro-modeling approach to model shear-compression failure in masonry. To achieve this, they extended the formulation devised by Attard and Tin-Loi (2005) for the numerical simulation of fracture in concrete. In that model, a masonry wall is modeled as a set of masonry units. Furthermore, each masonry unit is subdivided into triangular finite element units and to simulate fracture, a constitutive softening-fracture law at the boundary nodes along the sides of the triangular finite element units is taken into account. The advantage of this formulation is that the tracking of interacting and/or branching cracks is allowed without remeshing.

3.2.2 Discrete Element (DE) Models

The discrete element (DE) method is characterized by modeling the materials as an assemblage of distinct blocks or particles interacting along their boundaries. According to its developers, Cundall and Hart (1989), the name “discrete” only applies to a computer approach if: (a) it allows finite displacements and rotations of discrete bodies and (b) new contacts between the blocks or particles are automatically recognized and updated as the calculation progresses. The formulation of the method was proposed initially by Cundall(1971) for the study of jointed rock, modeled as an assemblage of rigid blocks. Later this approach was extended to other fields of engineering requiring a detailed study of the contact between blocks or particles such as soil and other granular materials (Ghaboussi and Barbosa 1990). More recently the approach was applied successfully to model historic masonry structures and monuments by Sarhosis et al. (2014); Lemos (1995); Sincraian and Lemos (1999); Azevedo et al. (2000); Alexandris et al. (2004); Komodromos et al. (2008) and Papaloizou and Komodromos (2009 and 2012) in which the collapse modes were typically governed by mechanisms in which the deformability of the blocks plays little or no role. Also, the possibility of frequent changes in the connectivity and the type of contact as well as marked non-linearity induced by the inability of the masonry joints to withstand tension makes the usage of DE models suitable for solving problems involving discontinuities. The Distinct Element Method (DEM), the Finite-Discrete Element Method (FDEM), the Discontinuous Deformation Analysis (DDA) and the Particle Flow Code (PFC) are all different formulations of the DE method with important applications to masonry structures. These are considered in more detail below.

(a) Distinct Element Method (DEM)

The distinct element method is an explicit discrete element method based on finite difference principles, derived from Cundall’s original work. It is presented in the codes UDEC (Universal Distinct Element Code) and 3DEC, developed for commercial use by ITASCA Ltd for either the static or dynamic analysis of 2D and 3D problems of blocky systems(ITASCA 2004a, 2004b) and the codes PFC 2D and PFC 3D for particle flow simulations for granular material problems (ITASCA 2004c). In the distinct element method masonry bricks or blocks are represented as an assembly of rigid or deformable blocks that may take any arbitrary geometry. Rigid blocks do not change their geometry as a result of any applied loading. Deformable blocks are internally discretized into finite difference triangular or finite element zones. These zones are continuum elements as they occur in the Finite Element Method (FEM). However, unlike FEM, in the distinct element method a compatible finite element mesh between the blocks and the joints is not required. Mortar joints are represented as *zero thickness interfaces* between the blocks. Representation of the contact between blocks is not based on joint elements, as occurs in the discontinuum finite element models (D-FEMs). Instead the contact is represented by a set of point contacts (see Figure 8) with no attempt to obtain a continuous stress distribution through the contact surface. The assignment of contacts allows the interface constitutive relations to be formulated in terms of the stresses and relative displacements across the joint. In the normal direction, the mechanical behavior of mortar joints is governed by Eq. (2):

$$\Delta\sigma_n = -JK_n \cdot \Delta u_n \tag{2}$$

where JK_n is the normal stiffness of the contact, $\Delta\sigma_n$ is the change in normal stress and Δu_n is the change in normal displacement. Similarly, in the shear direction the mechanical behavior of mortar joints is controlled by a constant shear stiffness JK_s using the following expression:

$$\Delta\tau_s = -JK_s \cdot \Delta u_s \tag{3}$$

where $\Delta\tau_s$ is the change in shear stress and Δu_s is the change in shear displacement. Stresses calculated at grid points along contacts are submitted to the Mohr-Coulomb failure criterion which limits shear stresses along joints. The following parameters are often used to define the mechanical behavior of the contacts (Figure 9): the normal stiffness (JKn), the shear stiffness (JKs), the friction angle (Jfric), the cohesion (Jcoh), the tensile strength (Jten) and the dilation angle (Jdil).

More general joint constitutive laws can be implemented in DEM models, including those referenced in the previous section about D-FEM models. Codes such as UDEC and 3DEC also allow the user to program his own models in C++ and build DLLs, which provides a powerful tool for research projects.

As with the FEM, the unknowns are the nodal displacements and rotations of the blocks. However, unlike FEM, the unknowns in the DEM are solved explicitly by differential equations from the known

Figure 8. Deformable blocks with contact points (Lemos 2007)

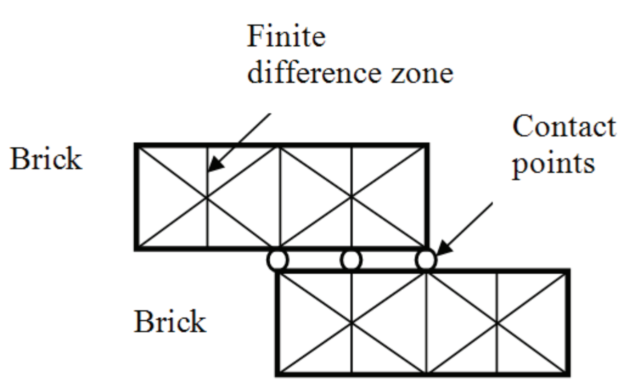
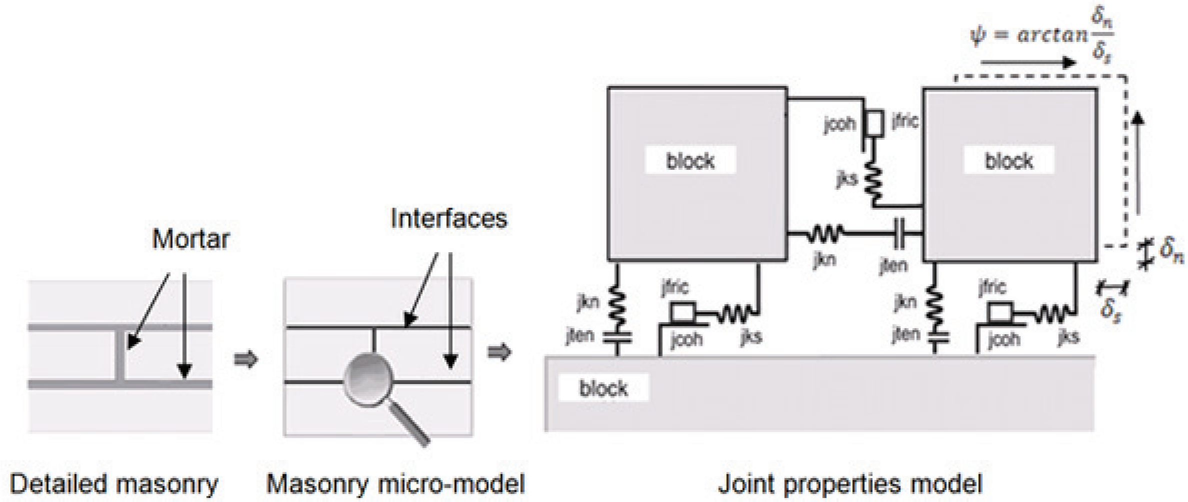


Figure 9. Interface model (Sarhosis 2012)



displacement, while Newton's second law of motion provides the motion of the blocks resulting from known forces acting on them. Therefore, large displacements and rotations of the blocks are allowed with the sequential contact detection and update of tasks automatically. This differs from the FEM where the method is not readily capable of automatically updating the contact size or creating new contacts. Mechanical damping is used in the DEM to solve both static and dynamic solutions. For static analysis, an approach similar to the dynamic relaxation technique is employed. In this technique, the equations of motion are damped to reach the equilibrium state as soon as possible (Asteris et al. 2013).

The choice between a rigid or deformable block representation depends on the type of structure and the relative importance of the deformation from the units and the joints. For static analysis, the computational overhead of using a coarse-mesh deformable block model is not very significant. However, for dynamic analysis, the larger time steps achievable with rigid block models are often a decisive factor (Lemos 2007b).

Usage of General Purpose DEM Software

Typical examples of distinct element analysis carried out for masonry structures using UDEC and 3DEC include:

a) Masonry wall panels: Dialer (1992); Rots (1997); Dialer (2002); Zhuge (2002); Schlegel and Rautenstrauch (2004a); Mohebkhahand Tasnimi (2007); Churilov and Dumova-Jovanoska (2008); Sarhosis et al. (2008); Sarhosis and Sheng (2014); Giamundo et al. (2014);
b) Dry stone retaining walls: Claxton et al. (2005); Walker et al. (2006);
c) Foundation movements in houses: Zhuge et al. (2004);
d) Narrow pier: Rots (1997);
e) Masonry-infilled steel frames with opening: Mohebkhah et al. (2008);
f) Ancient structures: Giordano et al. (2002); Lemos (2001); Roberti (2001); Azevedo et al. (2000);
g) Masonry buildings: Alexandris et al. (2004); Mohebkhah and Sarv-cheraghi (2014);

h) Stone masonry arches and aqueducts: Lemos (1995); Lemos (1997); Sincraian and Lemos (1999); Azevedo et al. (2000); Roberti (2001); Roberti and Calvetti(1998); Lemos (2004); Tóth et al. (2009); Sarhosis et al. (2014); and
i) Column-architrave structures underseismic action: Sarhosis et al.(2014); Papastamatiou et al. (1993); Papantonopoulos et al.(2002); Psycharis et al. (2000); Psycharis et al.(2003); Stefanou (2011).

For example, Sarhosis et al. (2014) carried out a numerical study to investigate the seismic vulnerability of the two story colonnade of the Forum in Pompeii (Figure 10), using the software UDEC. The colonnade was represented as an assemblage of distinct blocks connected together by zero thickness interfaces that could open and/or close depending on the magnitude and direction of stresses applied to them. Both static and non-linear static analyses had been undertaken. Also, a sensitivity study had been performed to investigate the effect of frictional resistance of the joints on the structural response of the colonnade. From the results of the push over analyses and by examining the capacity of this multi-drum colonnade, it was found that the UDEC is appropriate software to simulate the seismic behavior of masonry structures. Also, if the joint friction angle is 14 degrees, equilibrium of the structure cannot be granted and failure occurs.

In order to assess the suitability of the distinct element method for masonry, Zhuge (2002), carried out numerical studies with UDEC on shear wall panels with an opening, similar to those carried out by Lourenço (1996) using the discontinuum finite element method, see Figure 10. The results compare well with those obtained experimentally and when using the finite element method. UDEC managed to capture the tensile and shear failure of the joints as well as the final collapse mode, although it failed to predict the possible compressive failure at the bottom of the pier.

Furthermore, Mohebkhah (2007) carried out numerical studies with UDEC on dry joint stone masonry shear walls tested by Lourenço et al. (2005), in order to investigate the applicability of the distinct element method (DEM) for stone masonry shear walls analysis. Figure 11 compares the experimental and the numerical results of specimen SW.30 along with the discontinuous FEM results given in Lourenço et al.

Figure 10. Magnified deformed shape, seismic in-plane condition a_g=0.15g (Sarhosis et al. 2014)

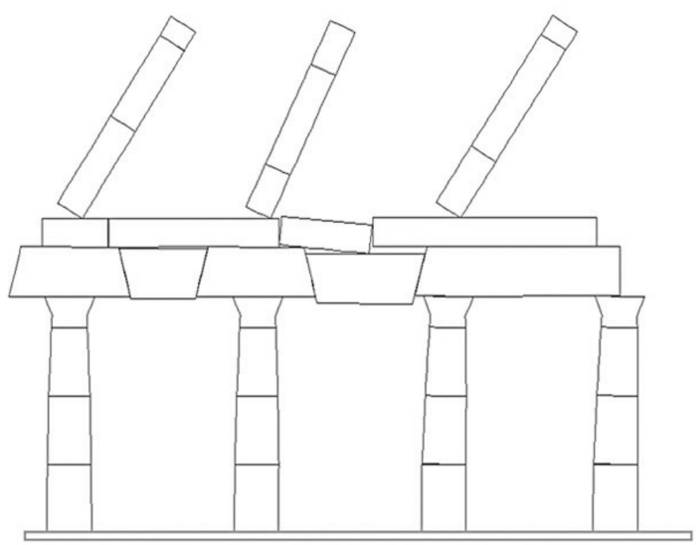
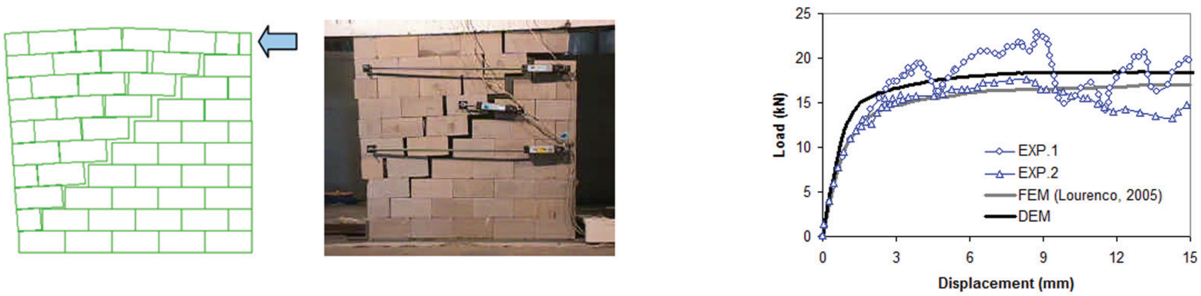


Figure 11.Comparison of experimental against numerical (FEM and DEM) results for dry joint masonry shear walls (Mohebkhah, 2007): (a) Computational and experimental crack pattern; (b) Load-displacement diagram



(a) **(b)**

(2005). The load-displacement curve obtained from the analysis with UDEC seems to be in good agreement with the experimental envelope. Also, it is seen that the pattern of diagonal cracking through bed and head joints predicted by DEM is reasonably consistent with the laboratory experimental observation.

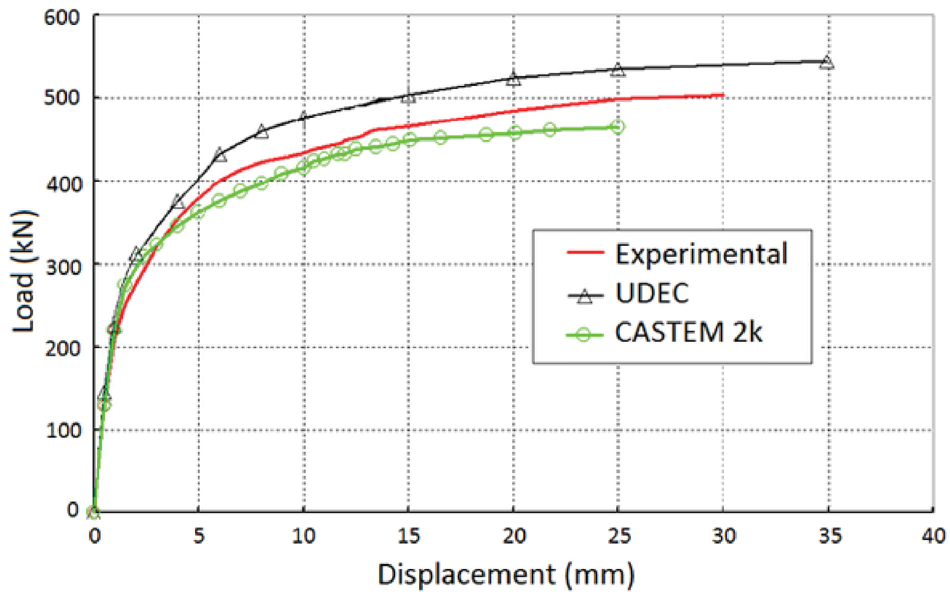
Furthermore, Giordano et al. (2002) investigated the applicability of the non-linear FEM (based on CASTEM 2000) and the Distinct Element method (based on UDEC) by comparing the computed results with those obtained from experimental tests on a full-scale masonry specimen forming part of the cloisters façade of the Sao Vicente de Fora monastery in Lisbon. Figure 12 compares the experimental and the numerical results for the initial monotonic load. The load-displacement curve obtained from the analysis with CASTEM seems to be in good agreement with the experimental envelope, although it is slightly stiffer compared to the experimental one. According to Giordano et al. (2002), such differences could be due to the simple modeling of the stone blocks and the masonry infill, meshed with poor-performing triangular elements, in which non-linearity is allowed. Also, the DE model reproduces well the general trend of the experimental envelope, even though the maximum load is slightly overestimated. The reason for this could be due to the simple modeling of the masonry in-fill by linear elastic blocks. In spite the limitations of each model, it is evident that both methods, to some extent, managed to grasp the global behavior of the experimental test and thus both of them can be used to model masonry. For the joint model implemented with the FEM, the main difficulty is the need to provide an easy methodology for re-meshing contacts when large displacements are allowed. Such a limitation is easily overcome by the DE method which is able to accommodate compatible meshes and large displacements. The main drawback identified for the DE model is the poor constitutive law for the internal elements when deformable blocks are taken into account.

Lemos (2001) performed a UDEC analysis of the first two complete cycles of the same ISPRA tests, which showed a fairly good comparison with the experimental results. The simple Coulomb friction model assigned to the joints was able to provide a reasonable match of the observed energy dissipation loops.

Development and Usage of Specialized DEM Software

Apart from the research works that are based on general-purpose DEM software, some researchers developed and used custom-made software based on the DEM. For example, Komodromos et al. (2008) developed a specialized software using the DEM to investigate the effect of certain parameters on the seismic behavior of ancient monumental structures with monolithic or multi-drum classical columns

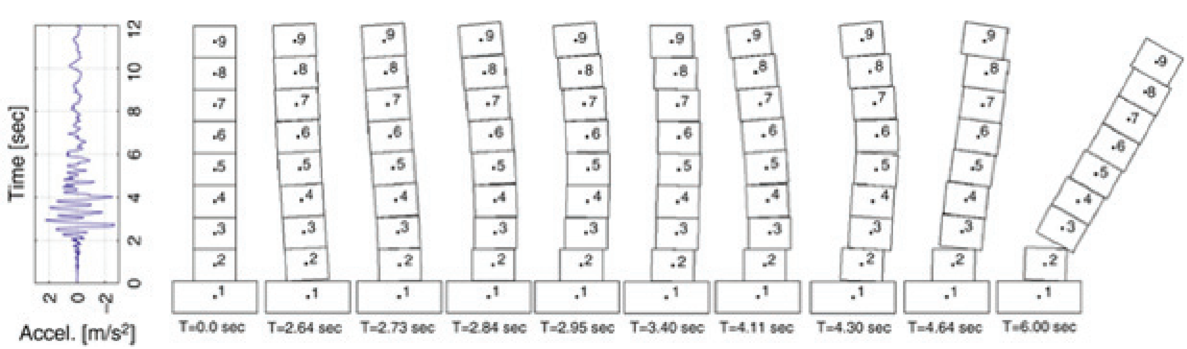
Figure 12. Comparison of experimental against numerical results (Giordano et al., 2002)



and colonnades (Figure 13). That study showed that multi-drum columns could be more vulnerable to collapse under excitations of low-frequency and their failure mode is governed by rocking mode. Papaloizou and Komodromos (2009) studied the influence of the frequency content and amplitude of the ground motions on the seismic response of columns and colonnades with epistyles. Papaloizou and Komodromos (2012) investigated the stability of multi-drum columns and colonnade systems with two rows of columns under strong earthquakes and showed that the required acceleration to overturn such structures decreases as the predominant frequency of the earthquake decreases.

The custom-made DEM software that was used for the aforementioned studies had been specifically designed and implemented to enable efficient performance of two-dimensional (2D) dynamic simulations under harmonic and seismic excitations of multi-block structures, while maintaining extensibility towards future spatial (3D) capabilities. Modern object-oriented design and programming approaches

Figure 13. Time-history response of a multi-drum column under the Kalamata Earthquake (Komodromos et al., 2008)



had been employed using Java technologies, in order to benefit from the significant advantages that these technologies offer to modern engineering software and enable the rapid development and extension of such complicated software (Komodromos et al. 2008).It is well known that the results obtained by 2D dynamic analysis of block assemblies are not capable of considering phenomena that may appear in the actual 3D response of such systems, such as off-plane movements and oscillations. Nevertheless, 2D analysis can be used to capture the overall phenomenon and various parameters that affect the seismic response of such structures requiring much less computational time. Therefore, 2D analysis can be used more efficiently and effectively when it is necessary to perform large numbers of simulations in order to study the effect of various parameters and characteristics, as 2D analysis is much more time efficient and much less sensitive to the selection of the values of the contact parameters.

In the developed software, the contact interactions were modelled using soft contact springs, which allows some interpenetration between the colliding bodies, and contact dashpots to evaluate the contact and damping forces, respectively, based on the interpenetration between bodies in contact. The interactions between two distinct bodies in contact are automatically generated as soon as a contact is detected, kept as long as the bodies remain in contact and removed as soon as the bodies are detached from each other. In order to be able to consider potential sliding according to the Coulomb law of friction, normal and tangential directions were considered during contact (Komodromos et al. 2008).

The analysis procedure of the developed software is presented in Figure 14. During the analysis, at each simulation step, all simulated bodies are checked with each other for contact. If no contact for a body is detected, gravitational, or any other surface or body forces are applied on the body. If contact is detected, the contact area and contact planes are determined and the relative velocities of the bodies in contact are computed. Using the contact springs and dashpots, the contact area and planes and the relative velocities, the contact forces are calculated and applied at each body, in addition to any other forces, such as gravitational forces, acting on the bodies.

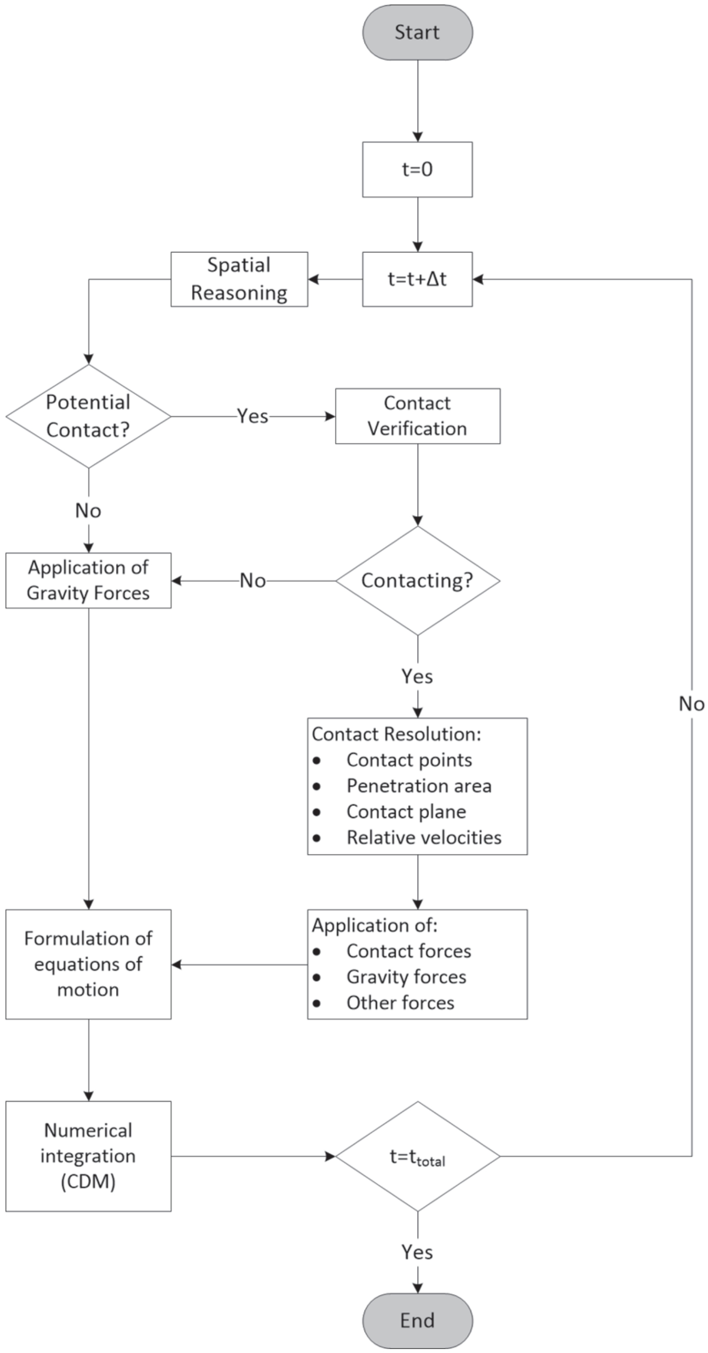
Taking into account all forces and moments applied at the centroid of each body, the equations of motion are formed and numerically solved using the Central Difference Method. After computing the displacements and rotations of all bodies, for each time step, their corresponding positions and orientations are determined. Then, a new cycle of contact detection, contact resolution, application of forces and numerical solution of the equations of motion follows, based on the updated positions of the bodies. This iterative procedure continues until the end of the simulation (Komodromos et al. 2008).

Usage of the DEM requires that the time step Δt is sufficiently small so that during a single step, disturbances cannot propagate between one discrete element and its immediate neighbors. Therefore, very small time steps are used, of the order of 10^{-6} sec, so as to satisfactorily capture the collisions and contacts among the individual discrete bodies of the simulated system.

(b) Discontinuous Deformation Analysis (DDA)

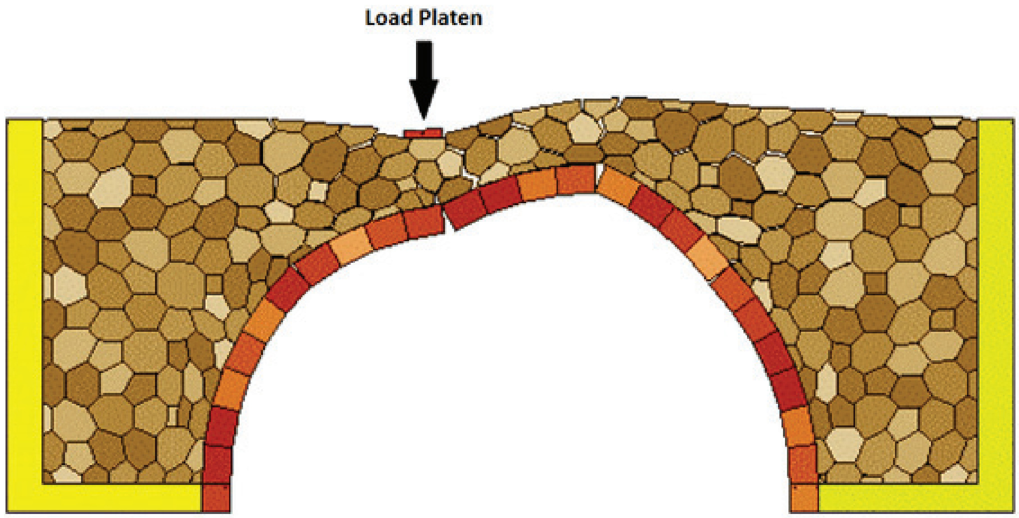
Discontinuous deformation analysis (DDA) is animplicit discrete element method developed by Shi and Goodman (1988) for solving stress-displacement problems for a jointed rock mass. Later, the method was applied successfully to model masonry arches by Ma et al. (1996); Bićanić et al. (2001) and Bićanić et al. (2003), see Figure 15.An application to masonry columns was presented byYagoda-Biran and Hatzor (2010).With DDA, the blocks are considered to be deformable but with a uniform stress and strain distribution. Contacts are considered rigid - the “hard contact approach” (Cundall and Hart 1989) and no interpenetration of blocks is permitted. This is opposed to the distinct element method where

Figure 14. Flow of control of the developed algorithms (Komodromos et al., 2008)



a "soft contact" approach is adopted and blocks can overlap when they are in compression. The hard contact approach is limited as shear displacement occurs only if the blocks slide. With the soft contact approach the contact forces are a function of the relative shear forces and sliding, which is more realistic when modeling masonry (Lemos 2007). Also, DDA uses an implicit algorithm based on a global stiffness matrix (unlike UDEC), for simultaneous solutions of the equations of equilibrium by minimizing

Figure 15. Deformed shape of an arch bridge using discontinuous deformation analysis with simplified deformable blocks (Bićanić et al. 2003)



the potential energy of the blocky rock mass system. Although DDA and the distinct element method (DEM) are similar in: a) the sense of representation of contacts; b) both can simulate the behavior of interacting discrete bodies; and c) they can recognize new contacts between bodies during calculations, they are quite different. Their fundamental differences are represented in Table 1, (Bićanić et al. 2001; Bobet et al. 2009). Moreover, DDA has two advantages over the DEM: permission for relatively larger time steps and closed-form integrations for the stiffness matrices of elements (Jing 2003).

Chiou et al. (1998, 1999) studied the static lateral load behavior of masonry-infilled RC frames using a refined discontinuous deformation analysis (DDA). In those studies, the masonry infill wall and RC frame were cut into sub-blocks by virtual joints with finite tension and shear strength of mortar and concrete materials, respectively. It was found that the DDA results are in good agreement with experimental results. However, the proposed model has some disadvantages such as: (1) ignoring the bond slip of reinforcement in the RC columns and beam, (2) ignoring mixed tension-shear failure of mortar joints

Table 1. Differences between DEM and DDA (Sarhosis 2012)

Distinct Element Method (DEM)	Discontinuous Deformation Analysis (DDA)
• Each block is discretized into the FE mesh; • Each block is treated separately during the analysis; • Stresses and forces are unknowns while displacements are computed from the stresses; • Contacts are resolved by defining the contact displacement and forces in terms of block overlap; • Uses an explicit procedure to solve the equilibrium equation; • Unbalanced forces drive the solution process and damping is used to dissipate energy.	• Each block has a uniform stress state; • The displacements are the unknowns; • Interpenetration of blocks is prevented by adding springs to the contacts; • Uses implicit method to achieve equilibrium; • As it is discontinuous analysis, it resembles and follows the procedures developed for FEM; • Total potential energy of the system is minimized to find the solution.

and concrete joints between concrete sub-blocks, (3) not taking into account the possibility of bricks' tensile failure in the infill wall regions with high normal stresses, and (4) considering linear constitutive relations for the bricks and concrete material (Chiou et al. 1998).

(c) Particle Models

Circular and spherical particle models belong to another type of discrete element methods that has been receiving increasing attention in various fields of engineering. The code PFC (Particle Flow Code, 2D or 3D) is a distinct element modeling software for micro-mechanical analysis of geomaterials and particulate systems. In this code, the granular microstructure of the material is modeled as a statistically generated assembly of rigid circular particles of varying diameters(Lisjak and Grasselli 2014).Yade is another particle-based code that has been recently developed by Kozicki and Donzé (2008 and 2009) to simulate numerically the behavior of geomaterials using discrete element method (Cundall and Strack 1979; Thavalingam et al. 2001; Tóth 2004; ITASCA 2004d).

Thavalingam et al. (2001) analyzed and compared the load-carrying capacity of an experimental backfilled masonry arch bridge (Figure 16) using a non-linear finite element technique with joint/interface elements (FEM based DIANA) and two models derived from the discrete element method (DDA and PFC). The load displacement curves obtained from the numerical simulations are shown in Figure 17. Of the three models, PFC seems to give better predictions of the collapse load obtained from the experimental study. The curves also show the ability of the discrete models to simulate the post peak structural response of the masonry arch bridge.

(d) Finite-Discrete Element Models (FDEMs)

As the name implies, discrete-finite element models are those that combine features of the DE and FE methods (Munjiza 2004). The method considers deformable blocks represented by a mesh of triangular elements which may split and separate during the analysis. Like the finite element technique, non-linear material models are used to define the characteristics of the masonry units and the mortar. Munjiza et

Figure 16. Model of a backfilled semi-circular masonry arch with PFC3D (Thavalingam et al. 2001)

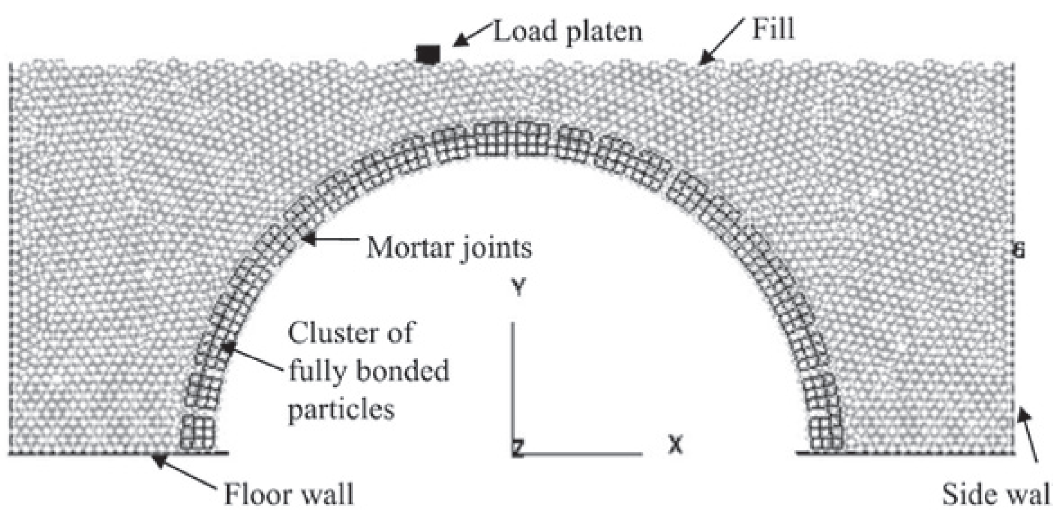
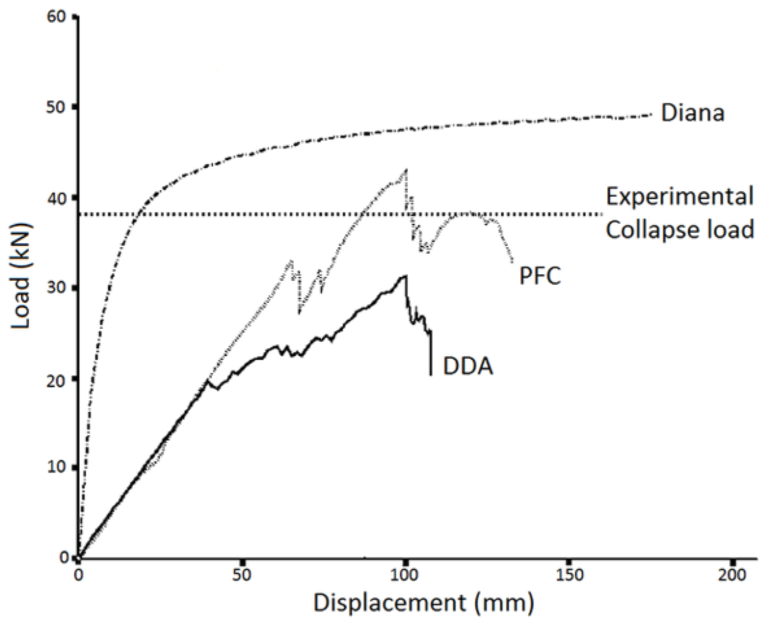


Figure 17. Comparison of experimental against numerical results (Thavalingam et al. 2001)



al. (1995) developed a method based on fracture mechanics criteria for simulating fracturing problems. Later, Mamaghani(1999) used a standard FE formulation for the block representation. Having assembled the global stiffness matrix, iterations are performed at each load increment to account for non-linear behavior. The method has been applied to the stability analysis of different masonry structures such as masonry arches and pyramids.

Also, Owen et al. (1998) used finite and discrete elements to assess the ultimate load carrying capacity of a damaged two span masonry arch bridge and to examine the efficiency of alternative repair strategies. Masonry blocks were represented by deformable discrete elements while the fill was represented by spherical discrete elements. The structure was subjected to abnormal vehicular loading in order to determine the load distribution through the backfill and into the arch rings. The numerical load–displacement prediction of a three ring arch bridge with an anchor stitching system was also presented, and compared with experimental results.

Furthermore, applications of the discrete finite element method have been used to design the strengthen measures for masonry arch bridges. Mullett et al.(2006) reported on the development of a model to assess the load-carrying capacity of a masonry arch bridge and to identify the suitability of different strengthening systems. The analysis was carried out using the explicit mode of the ELFEN (Rockfield Software Ltd.) package.

Finally, Komodromos and Williams (2004) and Komodromos (2005) proposed a simplified updated Lagrangian approach, which combined discrete and finite element methods, for the dynamic simulation of multiple deformable bodies, such as masonry structures. In particular, a combination of the DEM and an Updated Lagrangian (UL) FE formulation was proposed to be used together with the central difference method (CDM) and some simplifying assumptions that uncouple the contact interactions from the equations of dynamic equilibrium and the latter from each other. The problem is significantly simplified when small strains, linear elastic and isotropic material and diagonal mass and damping matrices are assumed and idealized contact springs are used, according to the soft contact approach, to consider the contact interactions in order to uncouple the problem. The CDM is used to explicitly integrate the equations of motion independently for each DOF, which avoids both the assemblage of matrices for the equations of motion and the need for an iterative numerical procedure. At the beginning of each simulation step the DEM are employed to identify the bodies in contact and determine the corresponding contact forces. First, an efficient contact detection algorithm is used to reduce the computational requirements and provide the necessary scalability. Then, for each newly identified contact between two bodies, the contact forces are computed using the corresponding contact springs and dashpots and the relative velocities and geometries of the bodies in contact. Each individual deformable body is discretized into an assemblage of finite elements and a FE formulation is used to describe the equations of motion. The contact forces are computed and applied together with all other forces to the simulated bodies. After the determination of the forces that act on each body, the equations of motion for the simulated bodies are solved using the CDM. Then, the stress and strain distributions of deformable bodies are evaluated from the computed displacements of each body.

The UL FE formulation (Bathe 1996) is derived from the principle of virtual work and the equations of motion can be decoupled and solved for each distinct body independently by replacing the unknown contact forces at each time step with their DEM estimations. According to the UL FE formulation all quantities and variables, such as displacements, strains and stresses, refer to the latest computed configuration at time t. Solving the equations of motion with the CDM, the displacements, strains and stresses at time $t+\Delta t$ can be computed and used to determine the new configuration of the simulated structural model. The force vectors are computed using the configuration of the simulated distinct bodies at time t, by integrating the product of the transpose of the strain displacement matrix with the Cauchy stresses over the deformed volume of the body at time t and summing over all elements. Because the displacements are large, instead of the usual relations for infinitesimal strains, the Almansi strains should be used, as they are work-conjugate strains of the Cauchy stresses (Komodromos 2005).

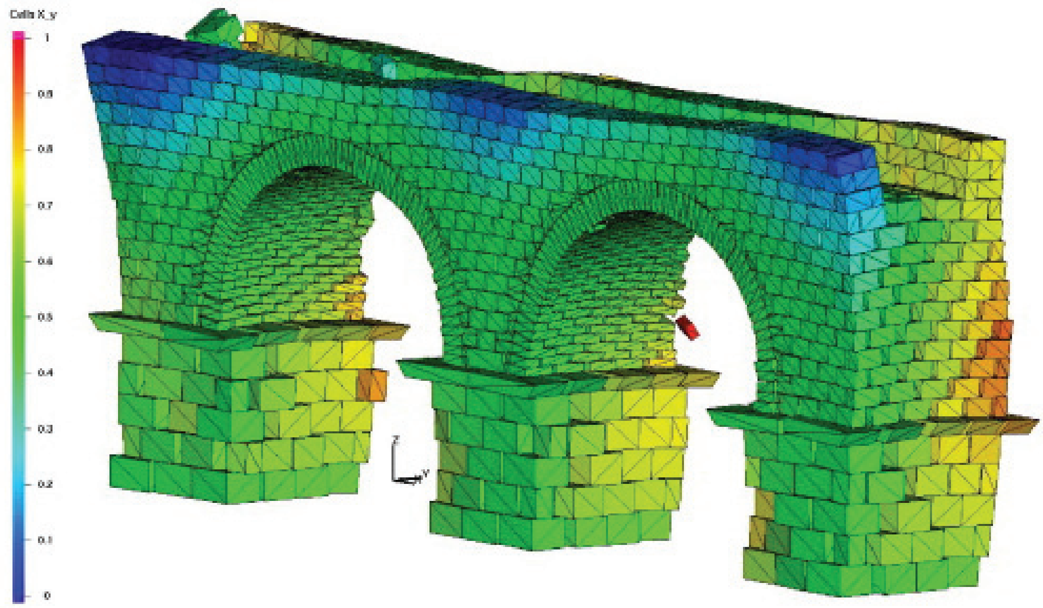
(e) Non-Smooth Contact Dynamics

The LMGC90® code based on the non-smooth contact dynamics (NSCD) has an implicit algorithm to solve the dynamic equations. An advantage of the NSCD method is that there is no need to resort to artificial damping in order to secure its numerical stability (Moreau 1998). Rafiee (2008) has used the NSCD method to model the behavior of the Arles aqueduct (Figure 18) as an assembly of 3D rigid blocks during the seismic excitation. From the results analysis, a high number of block detachments occurred during this short vibration. By considering the observed similarities between the simulated structure and the *in situ* one, conclusions drawn related to the destruction of the Arles aqueduct (circa 150 AD).

3.2.3 Boundary Element Models

Only a few studies deal with the application of the Boundary Element Method (BEM) in the modeling of masonry structures. Alessandri and Brebbia (1987) applied the BEM to the static analysis of two masonry walls subjected to experimental tests. The walls were brought to the point of collapse by applying horizontal loads and then, after being repaired by injections of cement mixtures, they were subjected to a new loading process culminating, as the first one, in the occurrence of the mechanism of collapse. The BEM results, in terms of displacement diagrams and internal stress states, were compared with the measurements and the crack patterns recorded during the experimental tests.

Figure 18. Application of the NSCD method to analyze the dynamic behavior of stone arched structures



Rashed et al. (1997) used the BEM to model the non-linear behavior of masonry. Cracking, debonding and crushing failure modes were considered, while the material non-linearity was ignored. Initial stresses, based on a developed algorithm, were used to represent the failure modes. The model used an incremental iterative solution procedure to track the failure at each loading stage. A masonry wall under vertical loading was analyzed using the model. The results were compared to the existing experimental and finite element results to show the accuracy and the validity of the model.

De Oliveira Neto et al. (2013) presented an application of a BEM formulation for anisotropic body analysis using isotropic fundamental solution. The anisotropy was considered by expressing a residual elastic tensor as the difference of the anisotropic and isotropic elastic tensors. Internal variables and cell discretization of the domain were considered. The formulation, the elastic tensor of the anisotropic medium properties and the algebraic procedure were presented. Two examples were presented to validate the formulation and good agreement was obtained when comparing analytical and numerical results. It was also shown that the proposed formulation exhibited close agreement between BE numerical results and different Finite Element (FE) models.

3.2.4 Discrete Limit Analysis Models (D-LAM)

Based on the pioneering work of Heyman (1969), Livesley (1978) first developed the limit analysis method for simulating discretized behavior in masonry structures. The following hypotheses are normally adopted:

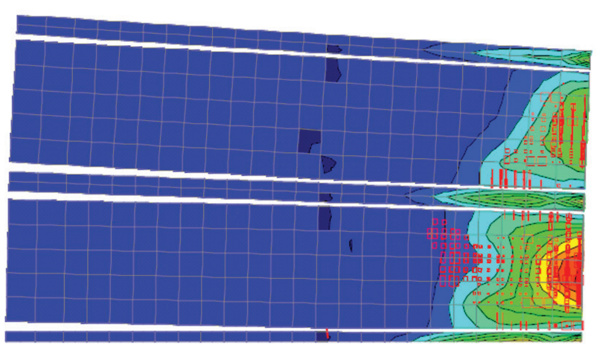
(a) The masonry units are infinitely rigid;
(b) The masonry units are infinitely strong;
(c) The masonry units do not slide at the joints;
(d) The joints transmit no tension.

According to Livesley's method, rigid blocks, typically larger than the physical blocks to account for the zero thickness interface, are analyzed using either the equilibrium or kinematic (work) form and are then solved using linear programming techniques (Ahmed and Gilbert 2003; Vanderbei 2001). Within non-standard limit analysis, block interfaces can be regarded as elements and the blocks as nodes. The equilibrium and kinematic compatibility equations are formulated in terms of finite sets of quantities (i.e. nodal displacements and forces or element stresses and strains). So, if the equilibrium form is formulated, the displacement and hinge rotations will become available. Many other researchers have applied limit analysis for the assessment of masonry structures (Gilbert and Melbourne 1994; Gilbert 2007; Orduña and Lourenço 2003).

3.3 Detailed Micro-Modeling

Detailed micro-modelling is able to represent the real behavior of each component of masonry. In this approach, both the elastic and inelastic properties of both the units and the mortar can be realistically taken into account. A suitable constitutive law is introduced in order to reproduce not only the behavior of the masonry units and mortar, but also their interaction. The major drawback of the method is that requires large computational effort to analyze. Today, this method is used mainly to simulate tests on small specimens in order to determine accurately the stress distribution in the masonry materials (Lourenço and Pina-Henriques 2006; Papa 2001; Rots 1991; Zucchini and Lourenço 2006). Adam et al. (2010) developed an FE model to understand the micromechanics of solid clay brickwork subjected to eccentric loads. The specimens were modelled by three-dimensional FE models using LUSAS 14.1. Bricks, mortar and brick mortar interface have been modelled assuming nonlinear constitutive equations with material parameter values based, in the first instance, on the tests performed on the materials and for those cases where test data is unavailable on an optimization procedure fitting the test data. In order to accurately investigate the micromechanics of the collapse mechanisms (i.e. the nonlinear, the cracking and crushing phenomena inside the brickwork) the FE models were three-dimensional. The numerical procedure has been verified against the results from previous laboratory tests (Figure 19).

Figure 19. Comparison between experimental and FEM failure patterns: (a) Computational crack pattern; (b) Experimental crack pattern



(a)

(b)

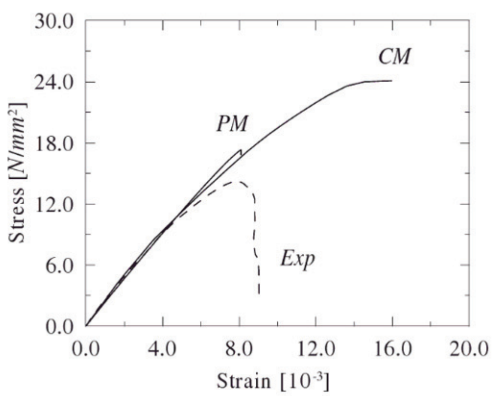
Lourenco (2010) used a polygonal particle based model consisting in a phenomenological discontinuum approach to represent the microstructure of units and mortars for the analysis of masonry assemblages under compression. The masonry components are composed by linear elastic particles of polygonal shape separated by non-liner interface elements. In this case, inelastic phenomena were able to occur in the interfaces while the process of fracturing consists of progressive bond-breakage. Typical numerical results together with experimental results are shown in Figure 19. Results are also compared with a continuum model (CM). The experimental results seem to be over-estimated with the continuum model, although a much better agreement observed with the experimental strength and peak strain with the particle model (PM) (Figure 20).

4. DISCUSSION ON THE APPROPRIATE MODELING APPROACH

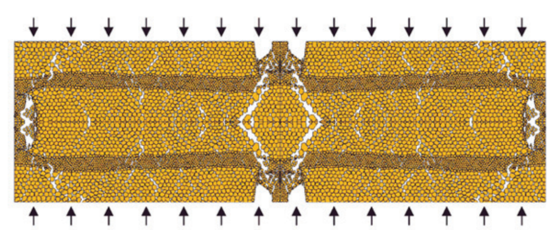
Numerical modeling approaches are divided into three distinct categories: detailed micro-modeling, simplified micro-modeling and continuum macro-modeling. Detailed micro-modeling is the most accurate approach to simulate the real behavior of structural masonry as both the masonry units and the mortar are discretized and modeled with continuum elements while the unit/mortar interface is represented by discontinuous elements accounting for potential crack or slip planes. However, due to the large computational effort required by *detailed micro-modeling*, it is used mainly to simulate the behavior of small scale masonry specimens and for research purposes.

The abovementioned drawback of the *detailed micro-modeling* can be partially overcome by the *simplified micro-modeling* approach. In this approach, the discontinuous masonry medium is represented as an assemblage of discrete units. The discontinuities are treated as boundary conditions between masonry units (e.g. bricks or blocks). In other words, the joints (consisting of mortar and the two unit-mortar interfaces) are viewed as interfaces between distinct units, while the units are slightly expanded in size in order to keep the masonry geometry unchanged. In this approach it is possible to simulate masonry as a set of elastic and inelastic blocks bonded together by potential fracture slip lines at the joints. The limitation

Figure 20. Comparison between experimental and numerical results of a masonry subjected in compression (Lourenco 2010): (a) experimental results compared to a standard continuum model (CM) and a particulate model (PM); (b) incremental deformed mesh at failure for the particle model



(a)



(b)

of the simplified micro-modeling approach is that the numerical problem quickly grows in complexity as the structure size increases. Furthermore, the drawback of the *simplified micro-modeling* approach is that the accuracy is lost to some extent, since the real joints are lumped into zero-thickness interfaces and hence, Poisson's effect on the mortar joints is ignored. In order to overcome the drawback, however, it has been suggested in the literature that the brick model should incorporate the compressive failure of masonry, which actually involves the interactive effects of both bricks and mortar, simultaneously.

Among the *simplified micro-modeling* approaches, it seems that the discrete element method is more suitable to simulate the response of discontinuous media like masonry structures subjected to either static or dynamic loading. In this case, large displacements and rotations between blocks, including sliding between blocks, the opening of the cracks and even the complete detachment of the blocks, and automatic detection of new contacts are allowed as the calculation progresses.

Unlike the abovementioned micro-modeling approaches, with the macro-modeling approach there is no distinction between individual masonry units and the mortar joints. Masonry is considered as a homogeneous anisotropic composite such that the joints and any cracks are smeared out in the continuum. Although this approach is very attractive for large-scale masonry structures, because of the reduced time and memory requirements as well as user friendly mesh generation, it is not adequate for detailed studies and for capturing failure mechanisms. It is shown that the modeling methods and the parameters must be carefully chosen as one method cannot be claimed suitable for all circumstances. The advantages and limitations of each method of computational modeling have been summarized in Tables 2 to 4.

5. COMMERCIAL SOFTWARE PACKAGES

There are a number of general-purpose finite element and discrete element commercial software packages available for numerical modeling and analysis of different types of structures. These packages are capable of performing two or three dimensional nonlinear static or dynamic analyses. Most of the packages developed based on the finite element method (e.g. ANSYS, ABAQUS, ADINA, DIANA, LUSAS, NASTRAN) include different kinds of elements, solution strategies and material behavioral modes and hence, have been used for the analysis of brittle materials, such as masonry (ANSYS: Bayraktar et al.

Table 2. Comparison of simplified micro-modeling approaches - Discontinuum finite element models

Capabilities
• Can give good predictions of structural behavior; • Suitable for small deformations of the unit/joint interface.
Limitations
• Computationally expensive when dealing with large structures; • Difficulty to mesh blocks and joints when complex block arrangements are modelled or 3D cases; • The increase of sophistication of the models has largely increased the number of parameters necessary for the material models but dilation effects are not included; • When large displacements occurring the interfaces it is not possible to provide easy re-meshing, update existing contacts and/or create new ones; • Local stress singularities at the corners of the masonry units often occur creating inaccuracies; • No warning of masonry unit overlap; • The user needs to incorporate the interface constitutive law into the FE model; • Pre-defined crack patterns are required.

Table 3. Comparison of simplified micro-modeling approaches - Discrete element models (Sarhosis 2012)

Capabilities
• Large displacements and rotations of the units including complete detachment are allowed; • Updating existing contacts and recognizing new contacts automatically; • Preventing units virtual locking by rounding the units corners; • Ability to mesh the blocks independently, without the need to match nodal points (as in FEM); • Suitable for parallel processing; • Same algorithm for static and dynamic analysis; • Capable of modeling post peak behavior; • Overlap of units is restricted and dilation effects are included.
Limitations
• Contact detection can be computationally demanding; • Poor performance in terms of stress distribution accuracy; • Computationally expensive when dealing with large structures or when the deformability of within the discrete elements is taken into account; • The explicit solution procedure in the distinct element method is conditionally stable. • Uncertainty on the appropriate values of the contact parameters.

Table 4. Comparison of simplified micro-modeling approaches - Limit analysis models

Capabilities
• Suitable for the occasional user and practicing engineers; • Able to simulate fairly well the structural behavior of masonry; • Reduced number of input material properties; • Easy to discuss and understand alternative solutions; • Quick method.
Limitations
• Number of blocks is limited by computer power; • As the complexity of the structure increases so does the time to obtain results; • Suitable only for dry joint masonry due to the assumption of zero tensile strength; dilation effects are not included; • The lack of customized packages increases the time to prepare the model. • Does not provide information on the ductility resources.

(2010); ABAQUS: Giordano et al. (2002);Aref and Dolatshahi (2013); Kaminski and Bien (2013);Casolo et al. (2014); Casolo and Sanjust (2007);Minaei et al. (2014);Agnihotriet al. (2013); ADINA: Paret et al. (2008);Mallardo et al. (2013);DIANA: Najafgholipour et al. (2013 and 2014);LUSAS: Adam et al. (2010); NASTRAN: Juhasova et al. (2002)).

Discrete element software packages include: ELFEN (Rockfield Software Ltd., 2004), UDEC, 3DEC and PFC (ITASCA Ltd.) as well as the open-source software Yade (Kozicki and Donzé 2008).ELFEN is a comprehensive explicit finite element based Discrete Element software tool provided by Rockfield Software Ltd, which has been mainly used to address problems in highly specialized areas of activity within geomechanics. In this software, material softening is simulated using a non-associated Mohr-Coulomb model with shear strength parameters while, material softening associated with fracturing is captured using fracture mechanics principles incorporated in the governing equations (Lisjak and Grasselli 2014).

UDEC (Universal Distinct Element Code) and 3DEC are two- and three-dimensional distinct element modeling software, respectively, for geotechnical analysis of rock, blocky structures and structural support. UDEC was initially developed for the study of jointed and fractured rock masses in 1971 by Cundall (1971). However, due to its capability to explicitly represent the motion of multiple intersecting discontinuities, it is also suitable for analyzing masonry structures in which a significant part of the deformation is due to relative motion between the blocks. So far, various UDEC and 3DEC applications to the static and dynamic analysis of masonry structures have been reported in the literature (Lemos (1995); Lemos (1997);Sincraian and Lemos (1999);Azevedo et al. (2000); Psycharis et al. (2000); Roberti (2001); Giordano et al. (2002);Papantonopoulos et al. (2002); Psycharis et al. (2003);Zhuge et al. (2004); Claxton et al. (2005); Walker et al. (2006);Mohebkhah and Tasnimi (2007); Mohebkhah et al. (2008); Sarhosis et al. (2008);Tóth et al. (2009);Stefanou (2011);Sarhosis and Sheng (2014); Mohebkhah and Sarv-cheraghi (2014)).

6. SUMMARY/CONCLUSION

The study of historic masonry structures is a challenging task due to the difficulties encountered in the description of the complex geometry, morphology, material heterogeneity, material properties characterization, material variation due to weathering and deterioration effects and complex loading conditions. In the case of historic structures, an understanding of the serviceability and ultimate limit state behavior and the development of strategies to improve their performance is of paramount importance for the preservation of the cultural heritage. Over the last thirty years, considerable research effort has been carried out to describe the mechanical behavior of masonry structures to different levels of accuracy. Several analytical and computational methods have been developed. These ranges from empirical calculations and simplified engineering approaches, i.e. limit analysis, to detailed modeling taking into account each block of the masonry structure separately, i.e. discrete element modeling. The complexity of the model increases with the need for high computational power. In addition, sophisticated methods of analysis require complex material parameters to be identified; which usually require a number of costly and tedious laboratory experiments and destructive testing. In most cases, such properties are impossible to be obtained (Sarhosis and Sheng 2014). One should be aware that validation and calibration of numerical models based on the comparison of experimental tests and/or analytical formulations is essential to ensure the reliability of the numerical models and their ability to predict the structural response of the structure under different loading and boundary conditions. In spite of the modern developments in computational analysis and software engineering, the study of historical buildings is still facing significant difficulties with respect to computational power, input data acquisition and limited natural representation of the methods. A combined experimental-numerical basis is the key to validate, extend and improve the existing methods for the analysis of the mechanical behavior of historic masonry structures.

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APPENDIX: ABBREVIATIONS

The following abbreviations are used in the manuscript:

- BEM: Boundary element method.
- DDA: Discontinuous deformation analysis.
- DEM: Discrete/distinct element method.
- D-FEM: Discontinuum Finite element method.
- D-LAM: Discrete limit analysis model.
- FDEM: Finite-discrete element method.
- FEM: Finite element method.
- LAM: Limit analysis method.
- PFC: Particle flow code.
- URM: Unreinforced Masonry.