

# Metamodel-based computational techniques for solving structural optimization problems considering uncertainties

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**ABSTRACT:** Uncertainties are inherent in engineering problems due to various numerical modeling “imperfections” and due to the inevitable scattering of the design parameters from their nominal values. Under this perspective, there are two main optimal design formulations that account for the probabilistic response of structural systems: Reliability-based Design Optimization (RBDO) and Robust Design Optimization (RDO). In this work both type of problems are briefly addressed and realistic engineering applications are presented. The optimization part of the proposed probabilistic formulations is solved utilizing efficient evolutionary methods. In both types of problems the probabilistic analysis is carried out with the Monte Carlo Simulation (MCS) method incorporating the Latin Hypercube Sampling (LHS) technique for the reduction of the sample size. In order to achieve further improvement of the computational efficiency a Neural Network (NN) is used to replace the time-consuming FE analyses required by the MCS. Moreover, various sources of randomness that arise in structural systems are taken into account in a “holistic” probabilistic perception by implementing a Reliability-based Robust Design Optimization (RRDO) formulation, where additional probabilistic constraints are incorporated into the standard RDO formulation. The proposed RRDO problem is formulated as a multi-criteria optimization problem using the non-dominant Cascade Evolutionary Algorithm (CEA) combined with the weighted Tchebycheff metric.

## 1 Introduction

The basic engineering task during the development of any structural system is, among others, to improve its performance in terms of constructional or life-cycle cost and structural behaviour. Improvements can be achieved either by using design rules based

on the experience of the engineer, or via an automated manner by using optimization methods that lead to optimum structural designs. Strictly speaking, optimal means that for the formulation considered, no better solution exists. Taking into account the complexity of a structural optimization problem it is obvious that finding the global optimum solution is not an easy task. In real world applications, if uncertainties have not been taken into account, the significance of the optimum solutions would be limited. This is because, although nearly perfect structural models can be simulated in a computing environment, real world structures always have imperfections or deviations from their nominal state defined by the design codes. The optimum that is obtained through the numerical simulation is never materialized in an absolute way and as a result a near optimal solution is always applied in practice. A formulation of a structural optimization problem that ignores the scattering of the various design parameters is defined as a deterministic one. A numerically feasible optimum design, according to the deterministic formulation, once applied in a real physical system, may lose its feasibility due to the unavoidable dispersion on the values of structural parameters (material properties, dimensions, loads, etc). This happens because the performance of the applied design may be far worse than expected.

In order to account for the randomness of the most important parameters that affect the simulation and the response of a structure, a different formulation of the optimization problem based on stochastic analysis methodologies has to be used. The recent developments on the stochastic analysis methods (Schuëller 2005), has stimulated the interest for the probabilistic optimum design of structures. Over the last decade efficient probabilistic-based optimization formulations have been developed in order to account for the various uncertainties that are involved in structural design. There are two distinguished design formulations that account for the probabilistic systems response: Robust Design Optimization (RDO) (see Messac and Ismail-Yahaya (2002), Jung and Lee (2002), Doltsinis and Kang (2004), Lagaros and Papadrakakis (2006), among others), while detailed literature overview on RDO problems can be found in the work of (Park et al. 2006), and Reliability-based Design Optimization (RBDO) (see Frangopol and Soares (2001), Agarwal and Renaud (2004), Tsompanakis and Papadrakakis (2004), Youn et al. (2005), Agarwal and Renaud (2006), Ba-abbad et al. (2006) among others). RDO methods primarily seek to minimize the influence of stochastic variations on the nominal values of the design parameters. On the other hand, the main goal of RBDO methods is to design for minimum weight/cost, which satisfies the allowable probability of failure for certain limit state(s). In this study three characteristic probabilistic optimization problems of realistic steel structures are presented, in which efficient metamodels based on Neural Networks (NN) are incorporated in order to improve the computational efficiency of the proposed methodologies.

In all test examples considered, the randomness of loads, material properties, and member dimensions is taken into consideration using the Monte Carlo Simulation (MCS) method combined with Latin Hypercube Sampling (LHS). In order to deal with the increased computational cost required, despite the use of the LHS technique, by the MCS for lower limits of the probability of violation of the constraints, a NN-based methodology is adopted for obtaining computationally inexpensive estimates of the response required during the stochastic analysis. The use of NN is motivated by the approximate concepts inherent in stochastic analysis and the time consuming repeated analyses required for MCS. In each case a specially tailored NN is trained, utilizing

available information generated from selected conventional analyses. Subsequently, the trained NN is used to fast and accurately predict the output data for the next sets of random variables. It appears that the use of a properly selected and trained NN can eliminate any limitation on the sample size used for MCS and on the dimensionality of the problem, due to the drastic reduction of the computing time required for the repeated finite element analyses.

Firstly, the reliability-based sizing optimization of large-scale multistorey 3D frames is investigated. The objective function is the weight of the structure while the constraints are both deterministic (stress and displacement limitations) and probabilistic (the overall probability of failure of the structure). Randomness of loads, material properties, and member geometry are taken into consideration in the reliability analysis using Monte Carlo simulation. The probability of failure of the frame structures is determined via a limit elasto-plastic analysis. The optimization part is solved using Evolution Strategies (ES), while the limit elasto-plastic analyses required during the MCS are replaced by fast and accurate NN predictions.

Secondly, an efficient methodology is presented for performing RBDO of steel structures under seismic loading. Optimum earthquake-resistant design of structures using probabilistic analysis and performance-based design criteria is an emerging field of structural engineering. The modern conceptual approach of seismic structural design constitutes the so-called Performance-based Earthquake Engineering or PBEE (for details see the excellent book by (Bozorgnia and Bertero 2004)). An important ingredient of PBEE is structural reliability (Wen 2000): a straightforward consideration of all uncertainties and variabilities that arise in structural design, construction and serviceability in order to be able to calculate the level of confidence about the structure's ability to meet the desired performance goals. Due to the uncertain nature of the earthquake loading, structural design is often based on design response spectra of the region of interest and on some simplified assumptions on the structural behaviour under earthquake. In this test example the reliability-based sizing optimization of multistorey steel frames under seismic loading is investigated, in which the optimization part of RBDO is solved utilizing Evolution Strategies (ES) algorithm. The objective function is the weight of the structure, while the constraints are both deterministic (stress and displacement restrictions imposed by the design codes) and probabilistic (limitation on the overall probability failure of the structure which is defined in terms of maximum interstorey drift).

Finally, a hybrid Reliability-based Robust Design Optimization (RRDO) formulation is presented, where probabilistic constraints are incorporated into the standard RDO formulation. A similar RRDO formulation has been used in the work of Youn and (Choi 2004), where a performance moment integration method is proposed that employs a numerical integration scheme for output response to estimate the product quality loss. The proposed RRDO is formulated as a multi-criteria optimization problem using the non-dominant Cascade Evolutionary Algorithm (CEA) combined with the weighted Tchebycheff metric. The main goal of this approach is to account for the influence of probabilistic constraints in the framework of structural RDO problems, by comparing the RRDO formulation with the standard one. For this purpose, a characteristic test example of a 3D steel truss is investigated, where the objective functions considered in the RRDO formulation are the weight and the variance of the response of the structure, represented by a characteristic node displacement. During

the optimization process each structural design is checked whether it satisfies the provisions of the European design codes for steel structures (EC3 2003) with a prescribed probability of violation.

## 2 Formulations of probabilistic structural optimization problems

Generally, in structural optimization problems the aim is to minimize the weight of the structure under certain deterministic behavioural constraints usually imposed on stresses and displacements. The significant developments of stochastic analysis methods have stimulated the interest for their application in structural design resulting to two main categories of probabilistic optimum design formulations: Reliability-based Design Optimization (RBDO) and Robust Design Optimization (RDO). The main goal of RBDO methods is to achieve increased safety levels of the structure with respect to variations of the random design parameters, while RDO methods primarily seek to minimize the influence of stochastic variations on the mean design of a structural system. Since the aforementioned method can be complementary to each other, hybrid Reliability-based Robust Design Optimization (RRDO) formulations have also been presented, where probabilistic constraints are incorporated into the standard RDO formulation. There are also several other probabilistic optimization formulations, for example those based on convex set models, evidence theory, possibility theory, etc, which are described in other chapters of the present volume. In the sequence, the three aforementioned major types of stochastic optimization formulations are briefly described.

### 2.1 Reliability-based design optimization

In reliability-based optimal design additional probabilistic constraints are imposed in the standard deterministic formulation, in order to take into account various random parameters and to ensure that the probability of failure for the whole structure or some of its critical members is within acceptable limits. The probabilistic constraints enforce the condition that the probability of exceeding a certain limit state's threshold value is smaller than a certain value (usually from  $10^{-3}$  to  $10^{-5}$ ). Under this perspective, a discrete RBDO problem can be formulated in the following form:

$$\begin{aligned} \min \quad & C_{IN}(\mathbf{s}, \mathbf{x}) \\ \text{subject to} \quad & g_j(\mathbf{s}, \mathbf{x}) \leq 0 \quad j = 1, \dots, m \\ & p_f(\mathbf{s}, \mathbf{x}) \leq p_{all} \end{aligned} \quad (1)$$

where  $C_{IN}(\mathbf{s}, \mathbf{x})$  is the objective function (i.e. the structural weight or the initial construction cost) to be minimized,  $\mathbf{s}$  (which can take values only from the given discrete set  $R^d$ ) and  $\mathbf{x}$  are the vectors of the design and random variables, respectively. Regarding the constraints,  $g_j(\mathbf{s}, \mathbf{x})$  are the deterministic constraint functions and  $p_f(\mathbf{s}, \mathbf{x})$  is the probability of failure of the design that it is bound by an upper allowable probability equal to  $p_{all}$ . Most frequently, the deterministic constraints of the structure are the member stresses and nodal displacements or interstorey drifts. Furthermore, due to

engineering practice demands, the members are divided into groups having the same design variables. This linking of elements results in a trade-off between the use of more material and the need of symmetry and uniformity of structures due to practical considerations. Furthermore, it has to be taken into account that due to manufacturing limitations the design variables are not continuous but discrete since cross-sections belong to a certain set.

## 2.2 Robust design optimization

In practical applications, optimizing a single objective function, most often the material weight or cost, cannot capture every aspect related to the performance of the structure. Actually, in real world optimization problems, there are several conflicting and usually incommensurable criteria that have to be dealt with simultaneously. Such problems are called multi-objective or multi-criteria optimization problems. In addition, in the majority of cases the objective functions are conflicting and as a result there exists no unique point which represents the optimum for all of them. Consequently, the common optimality condition used in single-objective optimization must be replaced by a “multi-collective” concept, the so-called Pareto optimum. Thus, in the multi-criteria formulation of a robust design structural sizing optimization problem, implemented in this work, an additional objective function is considered which is related to the influence of the random nature of the structural parameters on the performance of the structure. The aim is to minimize both the weight and the variance of the response of the structure. The mathematical formulation of the RDO problem is as follows

$$\begin{aligned} \min \quad & [C_{IN}(s, \mathbf{x}), StDev_u(s, \mathbf{x})]^T \\ \text{subject to} \quad & g_j(s, \mathbf{x}) \leq 0 \quad j = 1, \dots, k \end{aligned} \quad (2)$$

where  $C_{IN}(s, \mathbf{x})$  is the initial construction cost and  $StDev_u(s, \mathbf{x})$  is the standard deviation of the response that correspond to the two objectives to be minimized,  $s$  and  $\mathbf{x}$  are the vectors of the design and random variables respectively and  $g_j(s, \mathbf{x})$  are the deterministic constraint functions.

## 2.3 Reliability-based robust design optimization

In a combined RRDO formulation the constraint functions can also vary, due to the random nature of the structural parameters. In the proposed RRDO formulation the probability of violation of the constraints is taken into account as an additional constraint function. The mathematical formulation of the RRDO problem implemented in this work is as follows

$$\begin{aligned} \min \quad & [C_{IN}(s, \mathbf{x}), StDev_u(s, \mathbf{x})]^T \\ \text{subject to} \quad & g_j(s, \mathbf{x}) \leq 0 \quad j = 1, \dots, k \\ & p_{v,max}(s, \mathbf{x}) \leq p_{all} \end{aligned} \quad (3)$$

where  $C_{IN}(s, \mathbf{x})$  is the initial construction cost and  $StDev_u(s, \mathbf{x})$  is the standard deviation of the response that correspond to the two objectives to be minimized,  $s$  and  $\mathbf{x}$  are

the vectors of the design and random variables respectively,  $g_j(\mathbf{s}, \mathbf{x})$  are the deterministic constraint functions, while  $p_{v,max}(\mathbf{s}, \mathbf{x})$  is the maximum probability of violation, among the  $k$  behavioural constraint functions, that it is bound by an upper allowable probability equal to  $p_{all}$ . In this study three types of deterministic behavioural constraints are imposed to the sizing optimization problem of the truss structure examined: (i) stress, (ii) compression force (for buckling) and (iii) displacement constraints. On the other hand, the employed probabilistic constraint enforces the condition that the probabilities of violation of certain limit state functions are smaller than a certain value.

### 3 Solving the optimization problem

As mentioned in the previous section, two types of optimization problems are encountered in the framework of this study: a single and a multi-objective one. Evolutionary based algorithms are employed for tackling both of them. The two most widely used optimization algorithms belonging to the class of evolutionary computation that imitate nature by using biological methodologies are the Genetic Algorithms (GA) and Evolution Strategies (ES). Initially the ES method was introduced in the seventies for mathematical type of optimization problems (see Schwefel 1981). In this work ES are used as the optimization tool for addressing demanding probabilistic optimization problems. Both GA and ES imitate biological evolution in nature and have three characteristics that make them differ from mathematical optimization algorithms: (i) instead of the usual deterministic operators, they use randomised operators, (ii) instead of a single design point, they work simultaneously with a population of design points, (iii) they can handle continuous, discrete and mixed optimization problems. The second characteristic allows for a natural implementation of ES on parallel computing environments (Papadrakakis et al. 1999).

Structural optimization problems have been mainly treated with mathematical programming algorithms, such as the sequential quadratic programming (SQP) method, which need gradient information. In structural optimization problems, and especially when uncertainties are considered, the objective function and the constraints are particularly highly non-linear functions of the design variables, thus the computational effort spent in gradient calculations is usually excessive. In studies by (Papadrakakis et al. 1999) and (Lagaros et al. 2002), it was found that probabilistic search methods are computationally more efficient than mathematical programming methods, even though more optimization steps are required in order to reach the optimum. In the former case the optimization steps are computationally less expensive than in the latter case since there is no need for gradient information.

#### 3.1 Solving the single objective optimization problem

The absence of sensitivity analysis in evolutionary methods has even greater importance in the case of probabilistic problems, since the calculation of the derivatives of the reliability constraints is very time-consuming. Furthermore, these methodologies can be considered, due to their random search, as global optimization methods because they are capable of finding the global optimum, whereas mathematical programming algorithms may be trapped in local optima. As it can be seen in Flowchart 21.1, the ES optimization procedure initiates with a set of parent vectors. If any of these

1. *Selection step*: selection of  $s_i$  ( $i=1, 2, \dots, \mu$ ) parent design vectors
2. *Analysis step*: perform structural analysis ( $i=1, 2, \dots, \mu$ )
3. *Constraints check*: all parent become feasible
4. *Offspring generation*: generate  $s_j$ , ( $j=1, 2, \dots, \lambda$ ) offspring design vectors
5. *Analysis step*: perform structural analysis ( $j=1, 2, \dots, \lambda$ )
6. *Constraints check*: if satisfied continue, else go to step 4
7. *Selection step*: selection of the next generation parent design vectors
8. *Convergence check*: If satisfied stop, else go to step 4

Flowchart 21.1 The ES algorithm for single-objective optimization problems.

parent vectors gives an infeasible design, then it is modified until it becomes feasible. Subsequently, the offspring design vectors are generated and checked if they are in the feasible region. According to the  $(\mu + \lambda)$  selection scheme, in every generation the values of the objective function of the parent and the offspring vectors are compared and the worst vectors are rejected, while the remaining ones are considered to be the parent vectors of the new generation. This procedure is repeated until the chosen termination criterion is satisfied.

### 3.2 Solving the multi-objective optimization problem

A number of techniques have been developed in the past, that adequately deal with the multi-objective optimization problem (Coello-Coello 2000, Mattson et al. 2004, Marler and Arora 2004). The multi-objective algorithm employed in this work belongs to the hybrid methods, where an evolutionary algorithm is combined with a scalarizing function. In general, when using scalarizing functions, locally Pareto optimal solutions are obtained. Global Pareto optimality can be guaranteed only when the objective functions and the feasible region are both convex or quasi-convex and convex, respectively. For non-convex cases, such as the majority of structural multi-objective optimization problems, a global single objective optimizer is required. In this work the non-dominant Cascade Evolutionary Algorithm using the augmented Tchebycheff metric (CEATm) is employed for the solution of the Pareto optimization problem at hand. This implementation was proposed by the authors in a previous work by (Lagaros et al. 2005), where more details of the present implementation can be found. The basic steps of the CEATm algorithm are outlined below in Flowchart 21.2, where it is obvious that the CEATm optimization scheme can easily be applied in two parallel computing levels, an external and an internal one. In addition, the multi-objective optimization problem is converted into a series of single objective optimization problems, where the solution of each subproblem can be performed concurrently.

## 4 Probabilistic analysis using Monte Carlo simulation

The reliability of a structure or its probability of failure is an important factor in the design procedure since it quantifies the probability under which a structure will fulfill

**Independent run  $i, i=1, \dots, nrun$** 

Generate/update the weight coefficients  $w_{ij}, j=1, \dots, m$  of the Tchebycheff metric.

**CEATm LOOP****1. Initial generation:**

1a. Generate  $\mathbf{s}_k$  ( $k=1, \dots, \mu$ ) vectors

1b. Structural analysis step

1c. Evaluation of the Tchebycheff metric

1d. Constraint check: if satisfied  $k=k+1$  else  $k=k$ . Go to step 1a

2. Global non-dominant search: Check if global generation is accomplished. If yes, then non-dominant search is performed, else wait until global generation is accomplished.

**3. New generation:**

3a. Generate  $\mathbf{s}_\ell$  ( $\ell=1, \dots, \lambda$ ) vectors

3b. Structural analysis step

3c. Evaluation of the Tchebycheff metric

3d. Constraint check: if satisfied  $\ell=\ell+1$  else  $\ell=\ell$ . Go to step 3a

4. Selection step: selection of the next generation parents according to  $(\mu + \lambda)$  or  $(\mu, \lambda)$  scheme

5. Global non-dominant search: Check if global generation is accomplished. If yes, then non-dominant search is performed, else wait until global generation is accomplished.

6. Convergence check: If satisfied stop, else go to step 5

END OF CEATm LOOP

**End of Independent run  $i$**

Flowchart 21.2 The CEATm algorithm for multi-objective optimization problems.

its design requirements. Structural reliability analysis, or probabilistic analysis is a tool that assists the design engineer to take into account all possible uncertainties during the design, construction phases and lifetime of a structure in order to calculate its probability of failure  $p_f$ , or probability of a limit state violation  $p_{viol}$ . In structural reliability analysis problems, the probability of violation of a limit state function, expressed as  $G(\mathbf{x}) < 0$ , can be written as

$$p_{viol} = \int_{G(\mathbf{x}) \geq 0} f_{\mathbf{x}}(\mathbf{x}) d\mathbf{x} \quad (4)$$

where  $\mathbf{x} = [x_1, x_2, \dots, x_M]^T$  is a vector of the random structural parameters and  $f_{\mathbf{x}}(\mathbf{x})$  denotes the joint probability of violation for all random structural parameters.

In probabilistic analysis of structures the Monte Carlo Simulation (MCS) method is very popular and particularly applicable when an analytical solution is not attainable. This is mainly the case in problems of complex nature with a large number of random variables where all other probabilistic analysis methods are not applicable. Despite its simplicity, MCS method has the capability of handling practically every possible case regardless of its complexity; it requires, though, excessive computational effort. In order to improve the computational efficiency of MCS, various techniques have been proposed.



Since MCS is based on the theory of large numbers ( $N_\infty$ ) an unbiased estimator of the probability of violation is given by

$$p_{viol} = \frac{1}{N_\infty} \sum_{j=1}^{N_\infty} I(\mathbf{x}_j) \quad (5)$$

in which  $\mathbf{x}_j$  is the  $j$ -th vector of the random structural parameters, and  $I(\mathbf{x}_j)$  is an indicator for successful and unsuccessful simulations defined as

$$I(\mathbf{x}_j) = \begin{cases} 1 & \text{if } G(\mathbf{x}_j) \geq 0 \\ 0 & \text{if } G(\mathbf{x}_j) < 0 \end{cases} \quad (6)$$

In order to estimate  $p_{viol}$  an adequate number of  $N$  independent random samples are produced. The value of the violation function is computed for each random sample  $\mathbf{x}_j$  and the Monte Carlo estimation of  $p_{viol}$  is given in terms of sample mean by

$$p_{viol} \cong \frac{N_H}{N} \quad (7)$$

where  $N_H$  is the number of successful simulations and  $N$  the total number of simulations.

In general, a vast number of simulations have to be performed in order to achieve great accuracy, especially for low values of probability of failure. In an effort to reduce the excessive computation cost of crude MCS using purely random sampling methodologies, which is considered as the drawback of the method, various sampling reduction techniques have been proposed. Among them are the importance sampling, adaptive sampling technique, stratified sampling, antithetic variate technique, conditional expectation technique, and Latin Hypercube Sampling (LHS), which was introduced by (MacKay et al. 1979). Although LHS is generally recognized as one of the most efficient size reduction techniques it has been proven to be efficient only in the case that relatively large probability of violation is to be calculated and in the case of the calculation of statistical quantities like the mean value and the standard deviation. In most other cases MCS-LHS performs like the crude MCS (Owen 1997).

In the LHS method, the range of probable values for each random variable is divided into  $M$  non-overlapping segments of equal probability of occurrence. Thus, the whole parameter space, consisting of  $N$  parameters, is partitioned into  $M^N$  cells. Then the random sample generation is performed, by choosing  $M$  cells from the  $M^N$  space with respect to the density of each interval, and the cell number of each random sample is calculated. The cell number indicates the segment number that the sample belongs to with respect to each of the parameters. Using LHS technique, sampling is realized independently, whereas, matching of random samples is performed either randomly, or in a restricted manner. All necessary random samples are produced and they are accepted only if they do not agree with any previous combination of the segment numbers. The advantage of the LHS approach is that the random samples are generated from all the ranges of possible values, thus giving a more thorough insight into the tails of the probability distributions.

## 5 NN-based MCS for stochastic analysis

Over the last ten years artificial intelligence techniques like neural networks (NNs) have emerged as a powerful tool that could be used to replace time consuming procedures in many engineering applications (Lagaros and Tsompanakis 2006), (Tsompanakis et al. 2007). Some of the fields where NNs have been successfully applied are: pattern recognition, regression (function approximation/fitting), optimization, nonlinear system modelling, identification, damage assessment, etc. Function approximation involves approximating the underlying relationship from a given finite input-output data set. Feed-forward NNs, such as multi-layer perceptrons (MLP) and radial basis function networks, have been widely used as an alternative approach to function approximation since they provide a generic functional representation and have been shown to be capable of approximating any continuous function with acceptable accuracy. A trained neural network presents some distinct advantages over the numerical computing paradigm. It provides a rapid mapping of a given input into the desired output quantities, thereby enhancing the efficiency of the structural analysis process. This major of a trained NN over the conventional procedure, under the provision that the predicted results fall within acceptable tolerances, leads to results that can be produced in a few clock cycles, representing orders of magnitude less computational effort than the conventional computational process.

In this work the application of NNs is focused on the simulation (i.e. probabilistic analysis of structures) of demanding computational problems of probabilistic mechanics. Many sources of uncertainty (material, geometry, loads, etc) are inherent in structural design and functioning. Probabilistic analysis of structures leads to safety measures that a design engineer has to take into account due to the aforementioned uncertainties. Probabilistic analysis problems, especially when earthquake loadings are considered, are highly computationally intensive tasks since in order to calculate the structural behaviour under seismic loads a large number of numerical analyses are required. In general, soft computing techniques are used in order to reduce the aforementioned computational cost. The aim of the present study is to train a neural network to provide computationally inexpensive estimates of analysis outputs required during the MCS process.

In the present work the ability of neural networks to predict characteristic measures that quantify the response of a structure considering uncertainties is presented. This objective comprises the following tasks: (i) select the proper training set, (ii) find suitable network architecture, and (iii) perform the training/testing of the neural network. The learning algorithm, which was employed for the training, is the well-known Back-Propagation (BP) algorithm (Rummelhart and (McClelland 1986). An important factor governing the success of the learning procedure of NN architecture is the selection of the training set. A sufficient number of input data properly distributed in the design space together with the output data resulting from complete structural analyses are needed for the BP algorithm in order to provide satisfactory results. Overloading the network with unnecessary similar information results to over training without increasing the accuracy of the predictions. The required training patterns are generated randomly using the LHS technique, where a parametric study is performed for defining the size of the training set for the efficient training of NN. The basic NN configuration

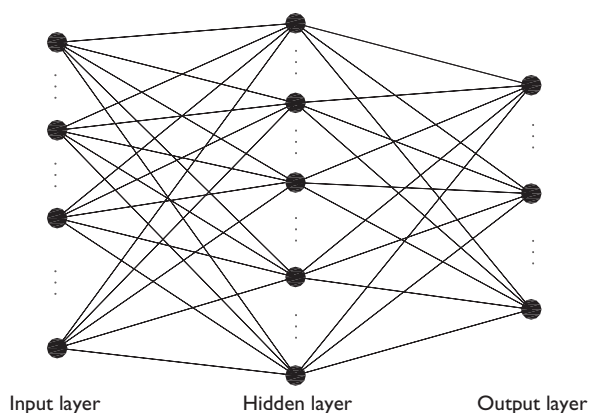


Figure 21.1 Typical neural network configuration.

employed for all the test cases examined in this study is selected to have one hidden layer, as shown in Figure 21.1.

## 6 Numerical results

### 6.1 RBDO of steel 3D frames under static loading using elasto-plastic analysis

Firstly, the reliability-based sizing optimization of multistorey 3D frame structures under static loading is investigated. The objective function is the weight of the structure while the constraints are both deterministic (stress and displacement limitations) and probabilistic (the overall probability of failure of the structure). Randomness of loads, material properties, and member geometry are taken into consideration in reliability analysis using the MCS method. The probability of failure of the frame structures is determined via a limit elasto-plastic analysis. The optimization part is solved using ES and two methodologies combining evolution strategies and neural networks (ES-NN) are examined.

In the first one, a trained NN utilizing information generated from a number of properly selected design vectors, computed by conventional finite element and reliability analyses, is used to perform both deterministic and probabilistic constraints checks during the optimization process. The data obtained from these analyses are processed in order to obtain the necessary input and output pairs which are subsequently used for training the NN. The trained NN is then applied to predict the response of the structure in terms of deterministic and probabilistic constraints checks due to different sets of design variables. The NN training is considered successful when the predicted values resemble closely the corresponding values of the conventional analyses which are considered exact. In the second methodology the limit elasto-plastic

analyses required during the MCS are replaced by NN predictions of the structural behaviour up to collapse. For every MCS that is required in order to perform the probabilistic constraints check, a NN is trained utilizing available information generated from selected conventional elasto-plastic analyses. The limit state analysis data are processed to obtain input and output pairs, which are used for training the NN. The trained NN is then used to predict the critical load factor due to different sets of basic random variables. A fully connected network, as shown in Figure 21.1, is used.

#### 6.1.1 Reliability-based structural optimization using MCS, ES and NN

In reliability analysis of elasto-plastic structures using MCS the computed critical load factors are compared to the corresponding external loading leading to the computation of the probability of structural failure. The probabilistic constraints enforce the condition that the probability of a local failure of the system or the global system failure is smaller than a certain value (i.e.  $10^{-5}$  to  $10^{-3}$ ). In this work the overall probability of failure of the structure, as a result of limit elasto-plastic analyses, is taken as the global reliability constraint. The probabilistic design variables are chosen to be the cross-sectional dimensions of the structural members and the material properties ( $E$ ,  $\sigma_y$ ).

MCS requires a number of limit elasto-plastic analyses that can be dealt independently and concurrently. This allows the natural implementation of the MCS method in parallel computing environment as well. The most straightforward parallel implementation of the MCS method is to assign one limit elasto-plastic analysis to every processor without any need of inter-processor communication during the analysis phase. In the present study the parallel computations were performed on a Silicon Graphics Power Challenge shared memory computer where the number ( $p$ ) of activated processors is equal to the number of the parent or offspring design vectors since  $\mu = \lambda$ .

#### 6.1.2 NN used for deterministic and probabilistic constraints check

In this methodology, a trained NN utilizing information generated from a number of properly selected design vectors is used to perform both the deterministic and probabilistic constraints checks during the optimization process. After the selection of the suitable NN architecture the training procedure is performed using a number ( $M$ ) of data sets, in order to obtain the I/O pairs needed for the NN training. The trained NN is then applied to predict the response of the structure in terms of deterministic and probabilistic constraint checks due to different sets of design variables.

The combined ES-NN optimization procedure is performed in two phases. The first phase includes the training set selection, the corresponding structural analysis and MCS for each training set required to obtain the necessary I/O data for the NN training, and finally the training and testing of a suitable NN configuration. The second phase is the ES optimization stage where the trained NN is used to predict the response of the structure in terms of the deterministic and probabilistic constraint checks due to different sets of design variables. This algorithm is summarized in Flowchart 21.3.

- NN training phase:
  1. *Training set selection step: select  $M$  input patterns*
  2. *Deterministic constraints check: perform the check for each input pattern vector*
  3. *Monte Carlo Simulation step: perform MCS for each input pattern vector*
  4. *Probabilistic constraints check: perform the check for each input pattern vector*
  5. *Training step: training of the NN*
  6. *Testing step: test the trained NN*
- ES-NN optimization phase:
  1. *Parents Initialization*
  2. *NN (deterministic-probabilistic) constraints check: all parents become feasible*
  3. *Offspring generation*
  4. *NN (deterministic-probabilistic) constraints check: if satisfied continue, else go to step 3*
  5. *Parents' selection step*
  6. *Convergence check*

Flowchart 21.3 The ES-NNI methodology.

### 6.1.3 NN prediction of the critical load in structural failure

In the second methodology the limit elasto-plastic analyses required during the MCS are now replaced by NN predictions of the structural behaviour up to collapse. For every MCS an NN is trained utilizing available information generated from selected conventional elasto-plastic analyses. The limit state analysis data is processed to obtain input and output pairs, which are used for training the NN. The trained NN is then used to predict the critical load factor due to different sets of basic random variables.

At each ES cycle (generation) a number of MCS are carried out. In order to replace the time consuming limit elasto-plastic analyses by predicted results obtained with a trained NN, a training procedure is performed based on the data collected from a number of conventional limit elasto-plastic analyses. After the training phase is concluded the trained NN predictions replace the conventional limit elasto-plastic analyses, for the current design. This algorithm is summarized in Flowchart 21.4.

### 6.1.4 Twenty-storey space frame RBDO example

A characteristic 3D building frame shown in Figure 21.2, has been tested in order to illustrate the efficiency of the proposed methodologies for reliability-based sizing optimization problems. The cross section of each member of the space frame considered is assumed to be a W-shape and for each structural member one design variable is allocated corresponding to a member of the W-shape data base. The objective function is the weight of the structure. The deterministic constraints are imposed on the interstorey drifts and for each group of structural members, on the maximum stresses due to axial forces and bending moments. The values of allowable axial and bending stresses are  $F_a = 150$  MPa and  $F_b = 165$  MPa, respectively, whereas the allowable interstorey drift is restricted to 1.5% of the height of each storey.

1. Parents Initialization
2. Deterministic constraints check: all parents become feasible
3. Monte Carlo Simulation step:
  - 3a. Selection of the NN training set
  - 3b. NN training for the limit load
  - 3c. NN testing
  - 3d. Perform MCS using NN
4. Probabilistic constraints check: all parents become feasible
5. Offspring generation
6. Deterministic constraints check: if satisfied continue, else go to step 5
7. Monte Carlo Simulation step:
  - 7a. Selection of the NN training set
  - 7b. NN training for the limit load
  - 7c. NN testing
  - 7d. Perform MCS using NN
8. Probabilistic constraints check: if satisfied continue, else go to step 5
9. Parents' selection step
10. Convergence check

Flowchart 21.4 The ES-NN2 methodology.

The probabilistic constraint is imposed on the probability of structural collapse due to successive formation of plastic hinges and is set to  $p_{all} = 0.001$ . The probability of failure caused by uncertainties related to material properties, geometry and loads of the structures is estimated using MCS with the LHS technique. External loads, yield stresses, elastic moduli and the dimensions of the cross-sections of the structural members are considered to be random variables. The loads follow a log-normal probability density function, while random variables associated with material properties and cross-section dimensions follow a normal probability density function.

The twenty-storey space frame shown in Figure 21.2 consists of 1,020 members with 2,400 degrees of freedom. This example is selected in order to show the efficiency of the proposed methodologies in relatively large-scale RBDO problems. The basic load of the structure is a uniform vertical load of 4.78 kPa at each storey and a horizontal pressure of 0.956 kPa acting on the x-z face of the frame. The members of the frame are divided into eleven groups, as shown in Figure 21.4, and the total number of design variables is eleven. The deterministic constraints are twenty-three, two for the stresses of each element group and one for the interstorey drift. The type of probability density functions, mean values, and variances of the random parameters are shown in Table 21.1. A typical load-displacement curve of a node in the top-floor is depicted in Figure 21.3, corresponding to the following design variables: 14WF176, 14WF158, 14WF142, 14WF127, 12WF106, 12WF85, 10WF60, 8WF31, 12WF27, 16WF36, 16WF36.

For this test case the  $(\mu + \lambda)$ -ES approach is used with  $\mu = \lambda = 10$ , while a sample size of 500 and 1,000 simulations is taken for the MCS-LHS. Table 21.2 depicts the performance of the optimization procedure for this test case. As can be seen, the probability of failure corresponding to the optimum computed by the deterministic optimization procedure is much larger than the specified value of  $10^{-3}$ . For this example the

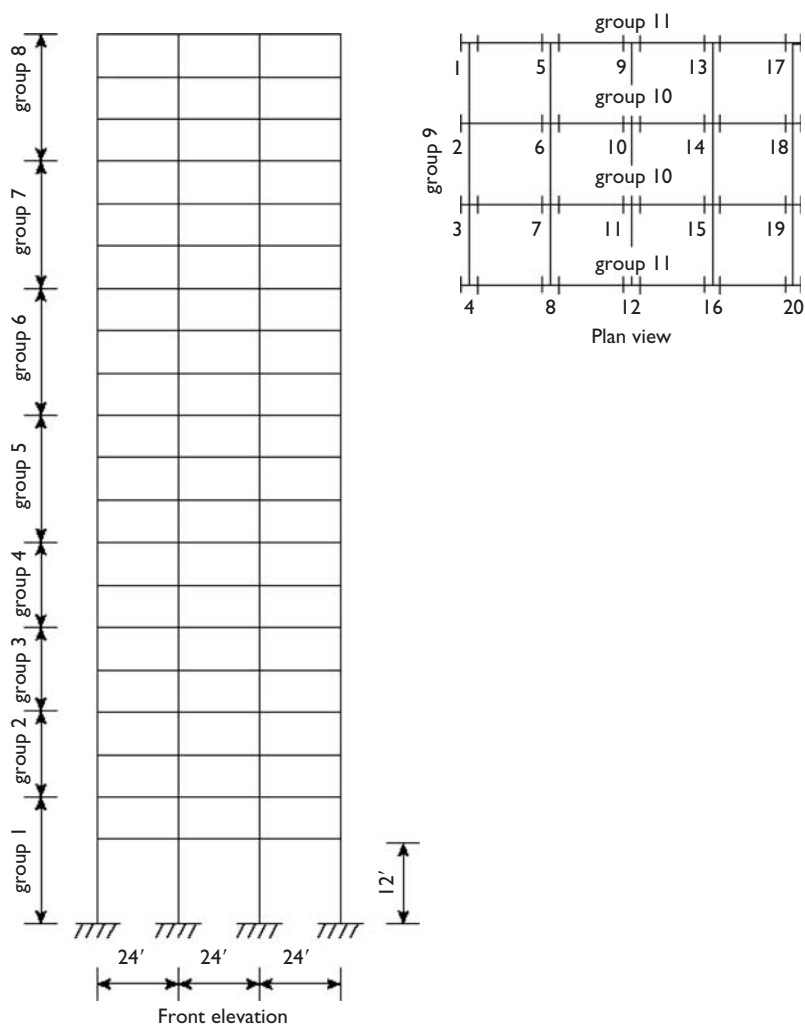


Figure 21.2 Description of the twenty-storey frame.

Table 21.1 Characteristics of the random variables.

Random variable	Probability density function (pdf)	Mean value	Standard deviation ( $\sigma$ )
E	N	200	0.10E
$\sigma_y$	N	25.0	0.10 $\sigma_y$
Design variables	N	$s_i$	0.1 $s_i$
Loads	Log-N	5.2	0.2

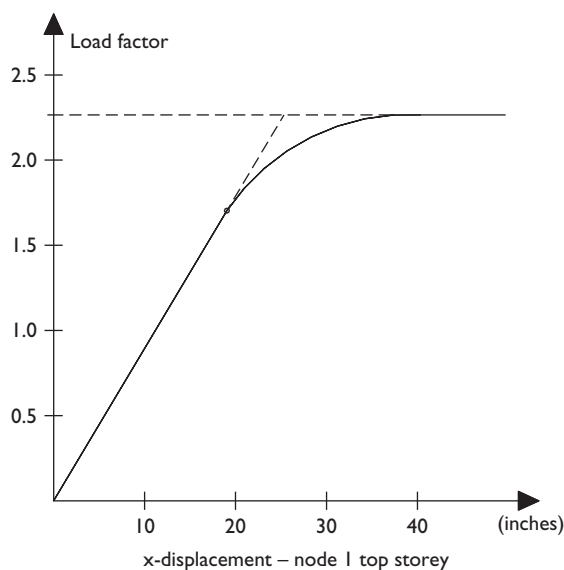


Figure 21.3 Load-displacement curve.

Table 21.2 Performance of the methods.

Optimization procedure	ES Gens.	$p_f^{**}$	Optimum weight (kN)	Sequential time (h)	Parallel time (h)		
					$p = 5$	$p = 10$	$p = 20$
DBO	83	$0.197 \cdot 10^{-0}$	6,771	2.0	0.7	0.3	0.3
RBDO (500 siml.)	126	$0.103 \cdot 10^{-2}$	9,114	141.0	28.4	14.1	7.1
RBDO-NN1 (500 siml.)	129	$0.102 \cdot 10^{-2}$	9,121	34.5	7.2	3.5	1.8
RBDO-NN2 (500 siml.)	126	$0.103 \cdot 10^{-2}$	9,114	15.8	3.3	1.7	0.9
RBDO (1,000 siml.)	120	$0.103 \cdot 10^{-2}$	9,156	250.3	50.1	25.1	12.6
RBDO-NN1 (1,000 siml.)	127	$0.101 \cdot 10^{-2}$	9,172	68.5	13.8	6.9	3.5
RBDO-NN2*	122	$0.97 \cdot 10^{-3}$	9,255	17.0	4.1	2.2	1.2

\*For 100,000 simulations.

\*\* For each final design and with 100,000 simulations using the NN2 scheme.

increase on optimum weight achieved, when probabilistic constraints are considered, is approximately 26% of the deterministic one, as can be observed in Table 21.2. In Table 21.2 showing the results of the test example, *DBO* stands for the conventional Deterministic-based Optimization approach, *RBDO* stands for the conventional Reliability-based Optimization approach, while *RBDO-NN<sub>i</sub>* corresponds to the proposed Reliability-based Optimization with NN incorporating algorithm *i* (*i* = 1, 2).

For the application of the RBDO-NN1 methodology the number of NN input units is equal to the number of design variables. Consequently, the NN configuration used in this case has one hidden layer with 15 nodes resulting in an 11-15-1 NN architecture which is used for all runs. The training set consists of 200 training patterns capturing



the full range of possible designs. For the application of the RBDO-NN2 methodology the number of NN input units is equal to the number of random variables, whereas one output unit is needed corresponding to the critical load factor. Consequently the NN configuration with one hidden layer results in a 3-7-1 NN architecture which is used for all runs. The number of conventional step-by-step limit analysis calculations performed for the training of NN is 60 corresponding to different groups of random variables properly selected from the random field. As can be seen in Table 21.2 the proposed RBDO-NN2 optimization scheme manages to achieve the optimum weight in one tenth of the CPU time required by the conventional RBDO procedure in sequential computing implementation. Table 21.2 also depicts the performance of the proposed methodologies in a straightforward parallel mode, with 5, 10 or 20 processors in which 5, 10 or 20 Monte Carlo simulations are performed independently and concurrently. It can be seen that the parallel versions of RBDO, RBDO-NN1 and RBDO-NN2 reached the perfect speedup irrespectively of the number of processors used.

The aim of the proposed RBDO procedure was threefold. To reach an optimized design with controlled safety margins with regard to various model uncertainties, while at the same time minimizing the weight of the structure and reducing substantially the required computational effort. The solution of realistic RBDO problems in structural mechanics is an extremely computationally intensive task. In the test example considered in this study the conventional RBDO procedure was found over seventy times more expensive than the corresponding deterministic optimization procedure. The goal of decreasing the computational cost by one order of magnitude in sequential mode was achieved using: (i) NN predictions to perform both deterministic and probabilistic constraints check, or (ii) NN predictions to perform the structural analyses involved in MCS. Furthermore, the achieved reduction in computational time was almost two orders of magnitude in parallel mode with the proposed NN methodologies.

## 6.2 RBDO of structures under seismic loading

In this section the reliability-based sizing optimization of multistorey framed structures under earthquake loading is investigated. The discrete RBDO problem is formulated in the form of Eq. (1), where  $C_{IN}(s, \mathbf{x})$  is the initial construction cost to be minimized,  $s$  and  $\mathbf{x}$  are the vectors of the design and random variables respectively,  $g_j(s)$  are the deterministic stress and displacement constraints. The overall probability of failure of the structure, as a result of multi-modal response spectrum analysis, is taken as the global reliability constraint. Failure is detected when the maximum interstorey drift exceeds a threshold value, here considered as 4% of the storey height, defined as  $p(\theta_{10/50} > \theta_{all})$  the probability that the drift  $\theta_{10/50}$  for the 10/50 hazard level exceeds the allowable drift  $\theta_{all}$ , that is bound by an upper allowable probability equal to  $p_{all}$ . For rigid frames with W-shape cross sections as in this study, the design constraints were taken from the design requirements specified by Eurocode 3 (2003) and Eurocode 8 (2004).

During the solution of the optimization problems a number of MCS runs are carried out for each different set of design variables. In order to replace the time consuming FE analyses by predicted results obtained with a trained NN, a training procedure is performed based on the data collected from a number of previously performed FE

analyses. After the training phase is concluded the NN predictions replace all conventional FE analyses, for the current design. For the selection of the suitable training pairs, the sample space for each random variable is divided into equally spaced distances. The central points within the intervals are used as inputs for the FE analyses.

The random variables considered are the cross-sectional dimensions of the structural members, the material properties ( $E$ ,  $\sigma_y$ ) and the loading conditions. Under the proposed approach the FE analyses required during the MCS are replaced by NN predictions of the structural response. For every design a NN is trained utilizing available information generated from selected conventional FE analyses. The trained NN is then used to predict the structural response for different sets of random variables depending on the type of problem examined.

### 6.2.1 Earthquake loading of steel frames

In Eurocodes earthquake loading is taken as a random action, therefore it must be considered for the structural design with the following loading combination:

$$S_d = \sum G_{kj} \text{ “+” } E_d \text{ “+” } \sum \psi_{2i} Q_{ki} \quad (8)$$

where “+” implies “to be combined with”,  $\Sigma$  implies “the combined effect of”,  $G_{kj}$  denotes the characteristic value of the permanent action  $j$ ,  $E_d$  is the design value of the seismic action, and  $Q_{ki}$  refers to the characteristic value of the variable action  $i$ , while  $\psi_{2i}$  is the combination coefficient for quasi permanent value of the variable action  $i$ , here taken as 0.30. Design code checks are implemented in the optimization algorithm as constraints. Each structural member should be checked for actions that correspond to the most severe load combination obtained from Eq. (8) and the permanent load combination:

$$S_d = 1.35 \sum G_{kj} \text{ “+” } 1.50 \sum Q_{ki} \quad (9)$$

It should be pointed out that the seismic action is obtained from the elastic spectrum reduced by the behaviour factor  $q$ . This is done because the structure is expected to absorb energy by deforming inelastically. Maximum values for the  $q$ -factor are suggested by design codes and vary according to the material and the type of the structural system. For the framed steel structures considered in this study  $q = 4.0$ .

The most common approach for the definition of the seismic input is the use of design code response spectra, a general approach that is easy to implement. However, if higher precision is required, the use of spectra derived from natural earthquake records is more appropriate. Since significant dispersion on the structural response due to the use of different natural records has been observed, these spectra must be scaled to the same desired earthquake intensity. The most commonly applied scaling procedure is based on the peak ground acceleration (PGA). Dynamic analysis of simple frames is most frequently performed using the multi-modal response spectrum analysis, which is based on the mode superposition approach and is briefly described in the next section.

### 6.2.2 Dynamic analysis using Multi-modal Response Spectrum

In general, the equations of equilibrium for a finite element system in motion can be written in the usual form

$$M\ddot{u}(t) + C\dot{u}(t) + Ku(t) = R(t) \quad (10)$$

where  $M$ ,  $C$ , and  $K$  are the mass, damping and stiffness matrices;  $R(t)$  is the external load vector, while  $u(t)$ ,  $\dot{u}(t)$  and  $\ddot{u}(t)$  are the displacement, velocity, and acceleration vectors of the finite element assemblage, respectively. A design approach based on the Multi-modal Response Spectrum (MmRS) analysis, which, in turn, is based on the mode superposition approach, has been used in the present study.

The MmRS method is based on a simplification of the mode superposition approach with the aim to avoid time history analyses which are required by both the direct integration and mode superposition approaches. In the case of the multi-modal response spectrum analysis Eq. (10) is modified according to the modal superposition approach to a system of independent equations

$$\bar{M}_i \ddot{y}_i(t) + \bar{C}_i \dot{y}_i(t) + \bar{K}_i y_i(t) = \bar{R}_i(t) \quad (11)$$

where

$$\bar{M}_i = \Phi_i^T M \Phi_i, \quad \bar{C}_i = \Phi_i^T C \Phi_i, \quad \bar{K}_i = \Phi_i^T K \Phi_i \quad \text{and} \quad \bar{R}(t) = \Phi_i^T R(t) \quad (12)$$

are the generalized values of the corresponding matrices and the loading vector, while  $\Phi_i$  is the  $i$ -th eigenmode shape matrix. According to the modal superposition approach the system of  $N$  differential equations, which are coupled with the off-diagonal terms in the mass, damping and stiffness matrices, is transformed to a set of  $N$  independent normal-coordinate equations. The dynamic response can therefore be obtained by solving separately for the response of each normal (modal) coordinate and by superposing the response in the original coordinates.

In the MmRS analysis a number of different formulas have been proposed to obtain reasonable estimates of the maximum response based on the spectral values without performing time history analyses for a considerable number of transformed dynamic equations. The simplest and most popular one is the Square Root of Sum of Squares (SRSS) of the modal responses. According to this estimate the maximum total displacement is approximated by

$$u_{\max} = \left( \sum_{i=1}^N u_{i,\max}^2 \right)^{1/2} \quad (13)$$

$$u_{i,\max} = \Phi_i y_{i,\max}$$

where  $u_{i,\max}$  corresponds to the maximum displacement vector corresponding to the  $i$ -th eigenmode.

Table 2 / 3 List of natural accelerograms.

	Earthquake name (Date)	Site\ Soil Conditions	Orientation	M <sub>S</sub>	PGA (g)	PGV (cm/sec)	d/v (sec)
1	Victoria Mexico (06.09.80)	Cerro Prieto\Alluvium	45	6.4	0.62	31.57	19.30
2	Kobe (16.01.95)	Kobe\Rock	0	6.95	0.82	81.30	9.91
3	Imperial Valley (19.05.40)	El Centro Array\CWB: D, USGS: C	180	7.2	0.31	29.80	10.32
4	Duze (12.11.99)	Bolu\CWB: D, USGS: C	90	7.3	0.82	62.10	12.99
5	San Fernando (09.02.1971)	Pacoima dam\Rock	164	6.61	1.22	112.49	10.69
6	Gazli (17.05.1976)	Karakyr\CWB: A	90	7.3	0.72	71.56	9.83
7	Friuli (06.05.1976)	Bercis\CWB: B	0	6.5	0.03	1.33	21.17
8	Aigion (17.05.90)	OTE building\Stiff soil	90	4.64	0.20	9.76	20.00
9	Central California (25.04.54)	Hollister City Hall\CWB: D, USGS: C	271	–	0.05	3.90	12.77
10	Alkyonides (24.02.81)	Korinthos OTE building\Soft soil	90	6.69	0.31	22.70	13.34
11	Northridge (17.01.94)	Jensen filter Plant\CWB: D, USGS: C	292	6.7	0.59	99.10	5.86
12	Athens (07. 09.99)	Sepolia (Metro Station)\Unknown	0	5.6	0.24	17.89	13.32
13	Cape Mendocino (25.04.92)	Petrolia\CWB: D, USGS: C	90	7.1	0.66	89.72	7.24
14	Erzihan, Turkey (13.03.92)	Erzikan East-East Comp\CWB: D, USGS: S	270	6.9	0.49	64.28	7.56
15	Kalamata (13.09.86)	Kalamata, Prefecture\Stiff soil	0	5.75	0.21	32.90	6.41
16	Iran (16.09.78)	Tabas\CWB: S	0	7.4	0.85	121.40	6.89
17	Loma Prieta <sup>1</sup> (18.10.89)	Hollister Diff Array\CWB: D	255	7.1	0.28	35.60	7.69
18	Loma Prieta <sup>2</sup> (18.10.89)	Coyote Lke dam\CWB: D	285	7.1	0.48	39.70	11.95
19	Mammoth Lakes (27.05.80)	McGee Creek\CWB: D	0	5	0.33	8.55	37.29
20	Irpinia, Italy (23.11.80)	Sturmo\Unknown	270	6.5	0.36	52.70	6.66

\*M<sub>S</sub>: Surface moment magnitude.

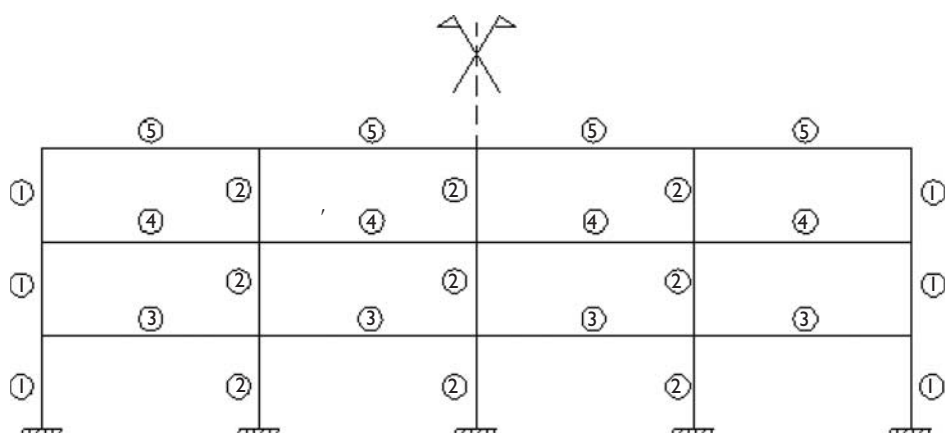


Figure 21.4 RBDO Test example – Geometry, member grouping.

### 6.2.3 Three-storey plane frame under earthquake loads RBDO example

One test example has been considered in the present study in order to illustrate the efficiency of the proposed methodology for reliability-based sizing optimization problems under earthquake loading. This test example is a four-bay, three-storey moment resisting plane frame shown in Figure 21.4. The frame has been previously studied by Gupta and Krawinkler (2000), where a detailed description of the structure is given. The frame consists of rigid moment connections and fixed supports. Each bay has a span of 9.15 m (30 ft), while each storey is 3.96 m (13 ft) high. The permanent action considered is equal to 5 kN/m<sup>2</sup> while the variable action is equal to 2 kN/m<sup>2</sup>, both distributed along the beams. The frame is considered to be part of a 3D structure where each frame is 4.5 m (15 ft) apart. The median spectrum used for the determination of the base shear corresponds to a peak ground acceleration of 0.32 g. Structural members are divided into five groups, as shown in Figure 21.4, corresponding to the five design variables of a discrete structural optimization problem. The cross-sections are W-shape beam and column sections available from manuals of the American Institute of Steel Construction (AISC). The objective function is the weight of the structure, to be minimized.

In this study a suite of twenty natural accelerograms, shown in Table 21.3, is used. It can be seen that each record corresponds to different earthquake magnitudes and soil conditions. The records of this suite comprise a wide range of PGA and peak acceleration over peak displacement ratio ( $a/v$ ) values. The latter parameter is considered to describe the damage potential of the earthquake more reliably than PGA. The records are scaled to the same PGA and their response spectra that are subsequently derived are shown in Figure 21.5. It has been observed that the response spectra follow the lognormal distribution. Therefore the median spectrum  $\hat{x}$ , also shown in Figure 21.5, and the standard deviation  $\delta$  are calculated from the above suite of spectra using the following expressions:

$$\hat{x} = \exp \left[ \frac{\sum_{i=1}^n \ln (Rd_i(T))}{n} \right] \quad (14)$$

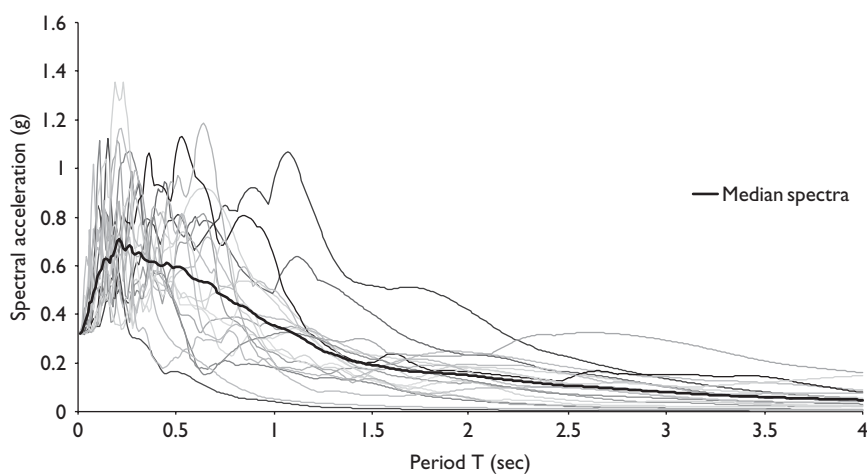


Figure 21.5 Natural record response spectra and their median.

Table 21.4 Characteristics of the random variables.

Random variable	Probability density function	Mean value	Standard deviation
E	N	2.1 10 <sup>6</sup> MPa	0.10E
$\sigma_y$	N	235 MPa	0.10 $\sigma_y$
Seismic load	Log-N	Median Spectrum (Eq. 14)	$\delta$ (Eq. 15)

$$\delta = \left[ \frac{\sum_{i=1}^n (\ln (Rd_i(T)) - \ln (\hat{x}))^2}{n - 1} \right]^{1/2} \tag{15}$$

where  $Rd_i(T)$  is the response spectrum value for period equal to  $T$  of the  $i$ -th record ( $i = 1, \dots, n$ , where  $n = 20$  in this study). For a given period value, the acceleration  $Rd$  is obtained as a random variable following the log-normal distribution with its mean value equal to  $\hat{x}$  and the standard deviation equal to  $\delta$ .

The deterministic constraints are related to stress and displacement constraints for steel frames according to Eurocodes. The probabilistic constraint is imposed on the probability of structural collapse which is set equal to  $p_{all} = 0.001$ . The probability of failure caused by uncertainties related to seismic loads and material properties of the structure is estimated using MCS with the LHS technique. The earthquake ground motion parameter, as described in Eq. (14), the yield stress and the elastic modulus are considered to be random variables. The type of probability density functions, mean values, and variances of the random parameters are shown in Table 21.4. The seismic action follows a log-normal probability density function, while the rest of

Table 21.5 Performance of the methods.

Optimization procedure	ES cycles	$p_f$	Time sequential (h)	Time parallel ( $p = 5$ ) (h)
DBO	157	0.0932	0.3	0.08
RBDO-MCS (5,000 siml.)	65	0.0008	557.3	140.1
RBDO-LHS (1,000 siml.)	72	0.001	149.1	40.6
RBDO-NN (100,000 siml.)	68	0.0009	42.1	16.2

the random variables follow a normal probability density function. For more details on probabilistic formulations of uncertainties the reader is referred to JCSS (2001) guidelines.

For this test case the  $(\mu + \lambda)$ -ES approach is used with  $\mu = \lambda = 5$  (equal to the number of design variables), while a sample size of 5000 simulations is taken. Table 21.5 depicts the performance of the optimization procedure for this test case. As it can be seen, the probability of failure corresponding to the optimum computed by the deterministic optimization procedure is much larger than the specified value of  $10^{-3}$ , thus unacceptable. On the other hand, the increase in safety results also in a significant increase on optimum weight. When probabilistic constraints are considered the weight increase is approximately 26% compared to the deterministic one, from 125.3 to 167.4 tn. Furthermore, the computation times are also enlarged in the case of RBDO, however, the use of NN as well as parallel computation reduces drastically the excessive computational cost of the process.

As far as the NN implementation is concerned, it was performed in a similar manner as the ES-NN1 algorithm that was described in the previous section. The NN configuration used has the typical architecture shown in Figure 21.1. It consists of three layers: one input, one hidden, and one output layer with varying number of nodes per layer. After an initial investigation on the optimum number of hidden layers and their nodes, one hidden layer was used having 10 nodes. The input data of the NN are the eleven random variables (two for each of the five element groups plus the seismic coefficient), while the output is one, i.e. the maximum interstorey drift value, which defines the limit-state violation. Thus, the NN configuration that was used was the following: 16-20-1. The training-testing set of the NN consisted of two hundred input/output pairs, twenty of which were used for testing the generalization capabilities of the trained NN. The application of NN reduces the computing time in a fraction of the time required for the conventional FE analyses. In addition, it does not affect the accuracy of the MCS method, in fact it can increase it since the fast NN approximations allow the use of much greater sampling size.

### 6.3 Hybrid RRDO 3D truss test example

For the purposes of this study a 3D steel truss structure has also been considered. For this test example, two objective functions have been taken into account, the initial construction cost and the standard deviation of a characteristic node displacement

representing the response of the structure. Two sets of constraints are enforced, deterministic constraints on stresses, element buckling and displacements imposed by the European design codes and probabilistic ones. Furthermore, due to manufacturing limitations the design variables are not continuous but discrete since cross-sections belong to a certain predefined set provided by the manufacturers. The discrete design variables are treated in the same way as in single optimum design problems using the discrete evolution strategies. The design variables considered are the dimensions of the members of the structure taken from the Circular Hollow Section (CHS) table of the Eurocode. The random variables related to the cross-sectional dimensions, for both test examples, are two per design variable: the external diameter  $D$  and the thickness  $t$  of the circular hollow section. Apart from the cross-sectional dimensions of the structural members, the material properties (modulus of elasticity  $E$  and yield stress  $\sigma_y$ ) and the lateral loads have also been considered as random variables. The robustness of the constraints is also considered using the overall probability of maximum violation of the behavioural constraints, as a result of the variation of the uncertain structural parameters.

The test example considered is the 3D truss tower shown in Figures 6(a) to 6(c). The height of the truss tower is 128 m, while its basis is a rectangle of side 17.07 m. The FE model consists of 324 nodes and 1254 elements which are divided into 12 groups that play the role of the design variables. The applied loading consists of: (i) self weight (dead load), (ii) live loads and (iii) wind actions according to the (Eurocode 1 2003). The type of probability density function, the mean value, and the variance of the random variables are shown in Table 21.6.

In the present implementation an investigation is performed on the ability of the NN to predict the required data for the evolution of the RRDO process. The inputs of the NN correspond to the random variables, while the outputs are the characteristic node displacement and the maximum displacement, stress and compression force required for the calculation of the probability of violation. The appropriate selection of I/O training data constitutes the most important parts in the NN training. The number of training patterns may not be the only concern, as the distribution of samples is of great importance also. Having chosen the NN architecture and trained the neural network, the probability of violation and the standard deviation of the response can be obtained in orders of magnitude less computing time. The modulus of elasticity, yield stress, diameter  $D$  and thickness  $t$  of the circular hollow cross-section as well as the loading have been considered as random variables for the structures examined. The inputs of the NN correspond to the random variables, while the outputs are the characteristic node displacement and the maximum displacement, stress and compression force required for the calculation of the probability of violation.

The previously described multi-criteria optimization (*CEATm*) algorithm employed is denoted as  $CEATm(\mu + \lambda)_{nruns, csteps}$  where  $\mu$ ,  $\lambda$  are the number of the parent and offspring vectors used in the ES optimization strategy,  $nruns$  is the number of independent CEA runs and  $csteps$  is the number of cascade stages employed. The basic steps inside an independent run of the multi-objective algorithm when the NN is embedded in the optimization process, as adopted in this test case, are described in Flowchart 21.5.

For the solution of the multi-objective optimization problem in question the non-dominant  $CEATm(\mu + \lambda)_{nrun, csteps}$  optimization scheme was employed, where  $\mu = \lambda = 5$ ,  $nrun = 10$  and  $csteps = 3$ . The resultant Pareto front curves for the RDO



**Independent run do  $i = 1, nrun$** 

## CEATm LOOP

**1. Initial generation:****do while**  $s_k$  not feasible  $k = 1, \mu$ Generate  $s_k$  ( $k = 1, \dots, \mu$ ) vectors

Analysis step

Evaluation of the Tchebycheff metric

Deterministic constraints check: if satisfied continue else regenerate  $s_k$  design

Monte Carlo Simulation step:

Selection of the NN training set

NN training for the limit load

NN Monte Carlo Simulations

Probabilistic constraint check

**end do****2. Global non-dominant search:** Check if global generation is accomplished. If yes, then non-dominant search is performed, else wait until global generation is accomplished.**3. New generation:****do while**  $s_\ell$  not feasible  $\ell = 1, \lambda$ Generate  $s_\ell$  ( $k = 1, \dots, \mu$ ) vectors

Analysis step

Evaluation of the Tchebycheff metric

Deterministic constraints check: if satisfied continue else regenerate  $s_k$  design

Monte Carlo Simulation step:

Selection of the NN training set

NN training for the limit load

NN Monte Carlo Simulations

Probabilistic constraint check

**end do****4. Selection step:** selection of the next generation parents according to  $(\mu + \lambda)$  or  $(\mu, \lambda)$  scheme**5. Global non-dominant search:** Check if global generation is accomplished. If yes, then non-dominant search is performed, else wait until global generation is accomplished.**6. Convergence check:** If satisfied stop, else go to step 3

END OF CEATm LOOP

**End do of Independent run**

Flowchart 21.5 The CEATm algorithm combined with NN.

Table 21.6 3D truss tower example: Characteristics of the random variables.

Random variable	Description	Probability density function	Mean value ( $\mu$ )	Standard deviation ( $\sigma$ )	$\sigma/\mu$	95% of Values interval
E (kN/m <sup>2</sup> )	Young's Modulus	Normal	2.10E+08	1.50E+07	7.14%	(1.81E+08, 2.39E+08)
$\sigma_y$ (kN/m <sup>2</sup> )	Allowable stress	Normal	355000	35500	10.00%	(2.85E+05, 4.25E+05)
F (kN)	Horizontal loading	Normal	$F_\mu$	0.4 $F_\mu$	40.00%	(2.16 $F_\mu$ , 17.84 $F_\mu$ )
D	CHS Diameter	Normal	$d_i^*$	0.02 $d_i$	2%	(0.9618 $d_i$ , 1.039 $d_i$ )
t	CHS Thickness	Normal	$t_i^*$	0.02 $t_i$	2%	(0.9618 $t_i$ , 1.039 $t_i$ )

\* Taken from the Circular Hollow Section (CHS) table of the Eurocode, for every design.

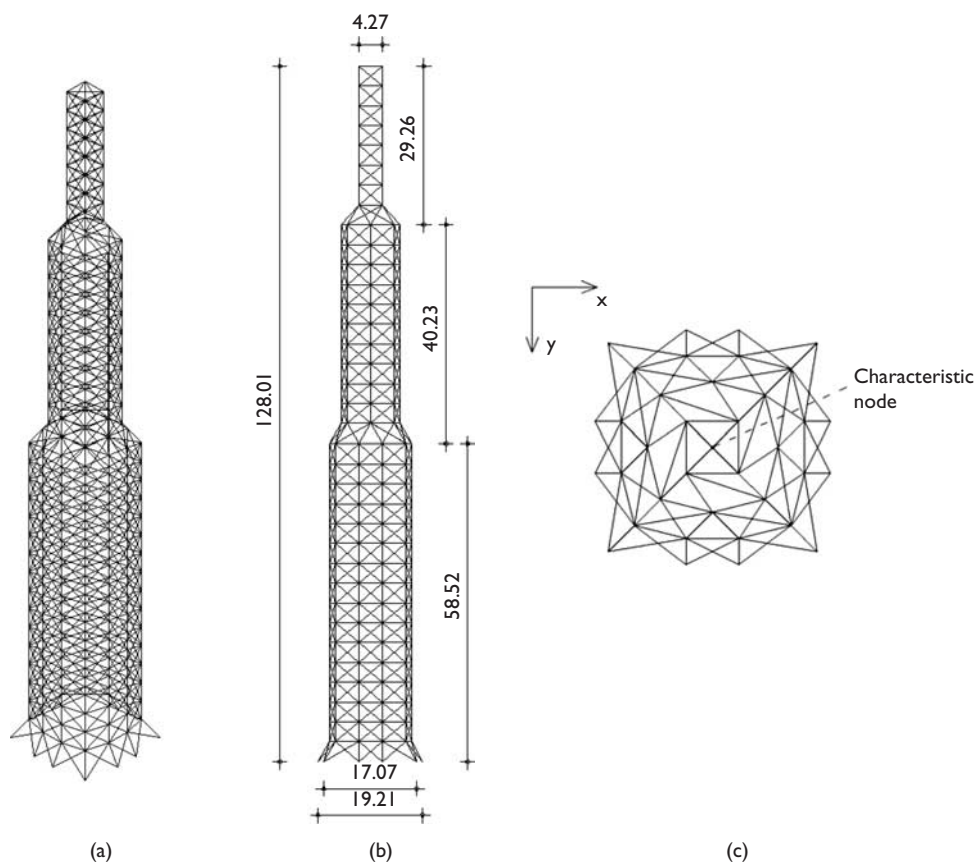


Figure 21.6 3D truss tower example: (a) 3D view, (b) Side view, (c) Top view.

formulations are depicted in Figure 21.7, with the structural weight on the horizontal axis and the standard deviation of the characteristic node displacement on the vertical axis. The displacement in the x-direction of the top node is selected as the characteristic one (Figure 21.6c). As can be seen in Figure 21.7 the trend on the influence of the probabilistic constraint is similar to that of the first example, where the Pareto front curves coincide in different parts.

Four different formulations of the RDO problem have been considered in this study: (i) the standard RDO formulation, (ii) RRDO with allowable probability equal to 2% denoted as RRDO\_2%, (iii) RRDO with allowable probability equal to 0.1% denoted as RRDO\_0.1% and (iv) RRDO with allowable probability equal to 0.01% denoted as RRDO\_0.01%. As can be seen in Figure 21.7 the presence of the probabilistic constraint influences the Pareto curves near the DBO area (designs  $A_i$ ,  $i = 1, \dots, 4$ ) of the Pareto front, where the weight of the structure is the dominant criterion. On the contrary, the four Pareto front curves almost coincide at the areas

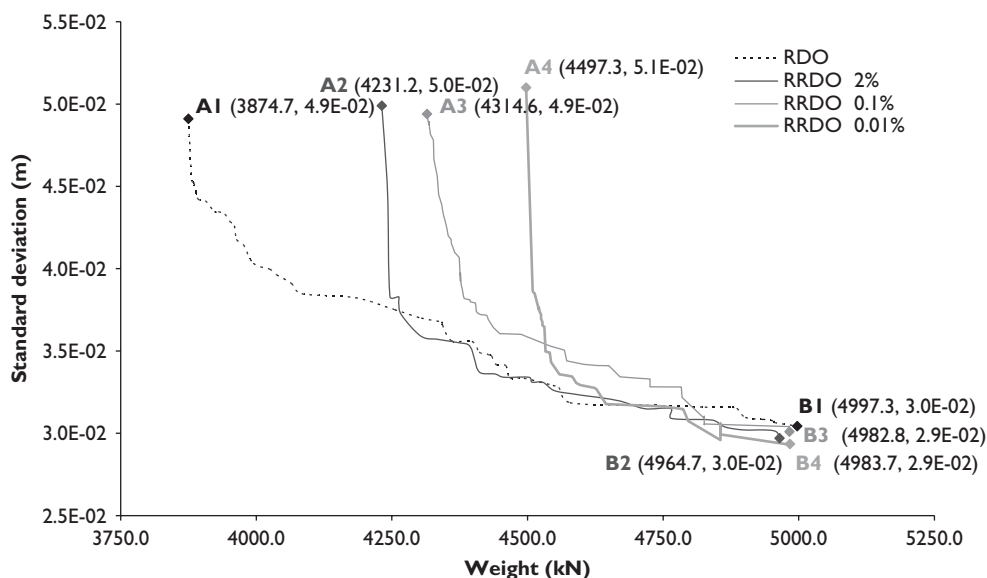


Figure 21.7 3D truss tower example: Comparison of the Pareto front curves.

where the importance of the second criterion (standard deviation of the response) increases.

The performance of the NN prediction is depicted in Figures 21.8a to 21.8d, where the prediction of the characteristic displacement, the maximum displacement, the maximum compressive force and the maximum tensile force are shown, respectively. Three different training sets, of size 100, 200 and 500, respectively have been examined, randomly generated using LHS, while 50 patterns have been used for testing. As can be seen in Figure 21.8, 100 samples are enough for efficiently training the NN. The MCS sample sizes used in this test example are 10,000, 100,000 and 500,000. In the RDO and RRDO\_2% formulations a sample size of 10,000 simulations has been used, while in RRDO\_0.1% and RRDO\_0.01% a sample size of 100,000 and 500,000 simulations has been employed. The different formulations and consequently the different sample sizes lead to a significantly different computing cost. In order to reduce the increased computing cost, especially of the last two formulations, a neural network formulation has been applied.

The NN configuration implemented in this example has one hidden layer with 50 nodes resulting in a 27-50-4 NN architecture (see Figure 21.1), which is used for all runs. The computing cost is depicted in Table 21.7, where the conventional and the corresponding NN computing times are reported. It has to be mentioned that the denoted basic computing costs for the RRDO\_0.1% and RRDO\_0.01% formulations are estimations due to the excessive computing cost required for these two cases. It can be seen that the NN-based methodology requires up to four orders of magnitude less computing time compared to the conventional one.

Table 21.7 3D truss tower example: Computing times.

Formulation	No of simulations	Time (hours)	
		Basic	NN
RDO	10,000	5.33E+01	5.87E-01
RRDO 2%	10,000	5.42E+01	5.96E-01
RRDO 0.01%	100,000	5.59E+02*	6.15E-01
RRDO 0.001%	500,000	2.79E+03*	6.14E-01

\* Estimated.

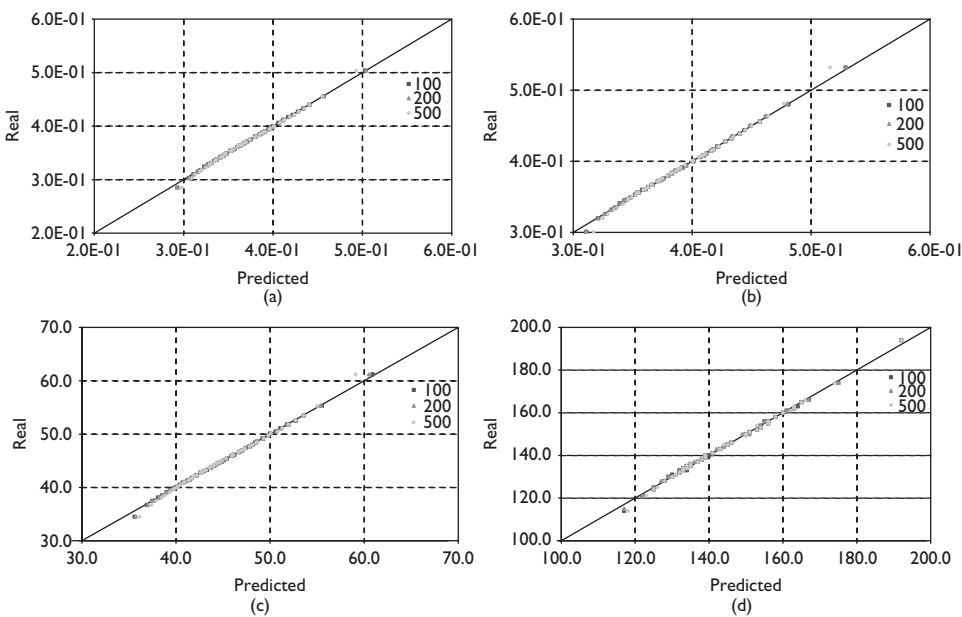


Figure 21.8 3D truss tower: Performance of NN with respect to the number of the training patterns (a) characteristic displacement, (b) maximum displacement, (c) maximum compressive force, and (d) maximum tensile force.

## 7 Conclusions

In most cases the optimum design of structures is based on nominal values of the design parameters and is focused on the satisfaction of the deterministically defined design code provisions. The deterministic optimum is not always a “safe” design, since there are many random factors that affect the design, i.e. manufacturing and performance of a structure during its lifetime. In order to find a “real” optimum the designer has to take into account all necessary random variables. In order to alleviate this deficiency, two types of formulations have been proposed in the past: RBDO and RDO. In the present work, apart from presenting successful RBDO applications, the combined RRDO is

also proposed, where probabilistic constraints are incorporated into the robust design optimization formulation.

In the examined RBDO formulations, under static or dynamic loads, the reliability analysis of the structure has to be performed in order to determine its optimum design taking into account a desired level of probability of structural failure. Only after forming and solving this RBDO problem, even with additional cost in weight and computing time, can a “global” and realistic optimum structural design be found. The aim of the proposed RBDO procedure is to increase the safety margins of the optimized structures under various uncertainties, while at the same time minimizing its weight, and reducing substantially the required computational effort. The solution of realistic RBDO problems in structural mechanics is an extremely computationally intensive task. As it can be observed from the numerical results, the computational cost for the solution of realistic RBDO problems is orders of magnitude larger than the corresponding cost for a deterministic optimization problem. Due to the size and complexity of RBDO problems, a non-conventional, stochastic evolutionary optimization method – such as ES – appears to be a suitable choice.

In a similar manner, in order to implement the hybrid RRDO formulation, structural reliability analyses for every candidate design have to be performed for the evaluation of the probability of violation. Depending on the value of the allowable probability of violation, different sample sizes are employed in order to calculate with sufficient accuracy the statistical quantities under consideration i.e. the standard deviation of the response and the probability of violation of the constraints. The Pareto front curves obtained for the presented RRDO formulation and the RDO formulation appear to be different when the weight objective function is predominant, while they approach each other in the areas of the Pareto fronts where the significance of the standard deviation of the response criterion increases. In other words, for the same standard deviation value, the optimum weight achieved by the RRDO formulation are larger than the corresponding weight achieved by the conventional RDO approach. Furthermore, it was observed that the presence of the standard deviation as an objective function forces the RDO formulation to produce results very close to those obtained by the RRDO formulation close to the right end of the Pareto front curve.

Concluding, the aim of this work was twofold: to examine the influence of the probabilistic parameters and constraints in structural optimization, and to deal with computationally demanding tasks in probabilistic mechanics. The computational effort involved in the conventional MCS becomes excessive in large-scale problems, especially when earthquake loading is considered, due to the enormous sample size and the computing time required for each Monte Carlo run. Although the LHS technique has been implemented for improving the computational efficiency of the MCS method, the computational cost remains excessive, making the solution of large-scale probabilistic optimization problems computationally unsolvable. Thus, a neural network assisted methodology has been proposed in order to obtain the structural response results required during the Monte Carlo simulations inexpensively. The achieved reduction in computational time was several orders of magnitude compared to the conventional procedure making tractable the optimization of real world structures under probabilistic constraints. The use of NN can practically eliminate any limitation on the scale of the problem and the sample size used for MCS without deteriorating the accuracy of the results.

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